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THE WHOLE Doctrine of Parallaxes EXPLAINED and ILLUSTRATED BY AN Arithemetical and Geometrical Conſtruction OF THE TRANSITS OF VENUS and MERCURY Over the SUN.

Enriched with a New and General Method of determing the Places where any Tranſit of theſe Planets may be beſt obſerved for the Inveſtigation of their Parallaxes; and applied in particular to that of Venus, which will be *June* 3, 1769, and of Mercury, *May* 7, 1799.

The SECOND EDITION,
Corrected and Enlarged.

By the Rev. Mr. EDWARD STONE, *A.M.*
Late Fellow of Wadham College, Oxford.

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TO THE RIGHT HONOURABLE

JAMES DOUGLAS,

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THE WHOLE DOCTRINE OF PARALLAXES

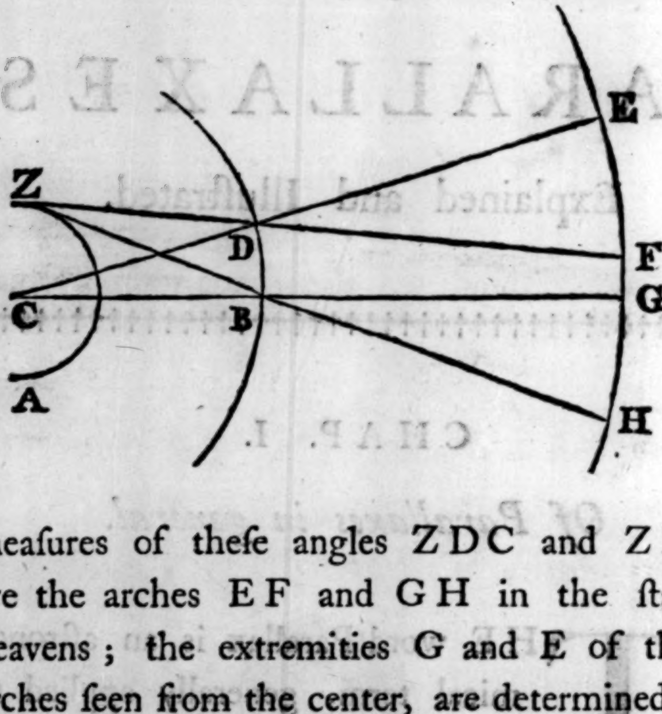
Explained and Illuſtrated.

CHAP. I.

Of Parallaxes in general.

1. THE word Parallax is an aſtronomical term, generally applied to the earth and ſome heavenly body; and in this application, it is the angle formed by two ſtrait lines meeting in the centre of that body, one of which is drawn from the centre of the earth, the other from any part of its ſurface.

Let ACZ be the earth, BD the orbit of any planet, B and D the places where it is ſeen at different times in the orbit from the center C, and any point Z upon the ſurface of the earth; the angles ZDC and ZBC, are the parallaxes of that body at thoſe places; the



meaſures of theſe angles ZDC and ZBC are the arches EF and GH in the ſtarry heavens; the extremities G and E of thoſe arches ſeen from the center, are determined by aſtronomical tables, the other extremities are found by obſervation.

2. It is ſelf evident that the nearer any planet is to the earth the larger is the parallactick angle; and the more remote, the leſs.

3. The

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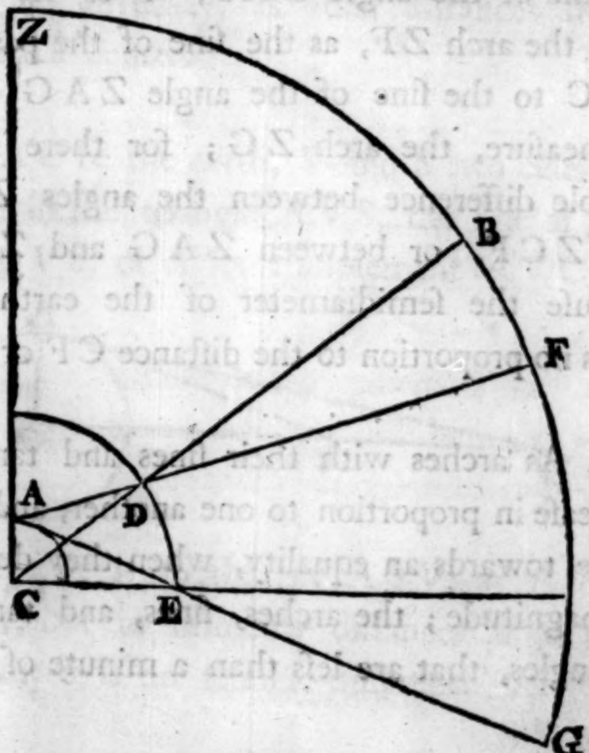
3. The parallax of the heavenly body that keeps the ſame diſtance from the center of the earth is greateſt in the horizon, and decreaſes the more that body is elevated above it.

4. The ſines of the parallaxes of the ſame body at different elevations above the horizon, are in proportion to the ſines of their arches from the zenith.

Let A C be the earth, Z the zenith, the angles A D C and A E C different parallaxes of the ſame body; then the Propoſition is, that



the ſine of the angle ADC is to the ſine of AEC, as the ſine of the arch FZ is to the ſine of the arch GZ.

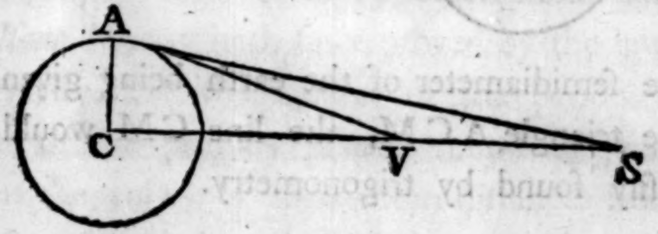
In the triangle ADC the ſine of the angle ADC is to the ſine of the angle CAD, or ZAF its complement to two right angles, as AC to CD, by a known trigonometrical rule; but the lines AC and CD are ſtanding lines, that is, they are the ſame in all parallaxes of this body, as in the triangle AEC: Therefore the ſine of the parallax ADC is to the ſine of the angle ZAF, or of its meaſure, the arch ZF, as the ſine of the parallax AEC to the ſine of the angle ZAG or of its meaſure, the arch ZG; for there is no ſenſible difference between the angles ZAF and ZCF, or between ZAG and ZCG, becauſe the ſemidiameter of the earth AC bears no proportion to the diſtance CF or CG.

5. As arches with their ſines and tangents decreaſe in proportion to one another, and converge towards an equality, when they decreaſe in magnitude; the arches, ſines, and tangents of angles, that are leſs than a minute of a de-

gree, have no ſenſible difference or diſproportion, and may be taken and uſed promiſcuouſly for one another: when therefore the parallactick angle is leſs than a minute of a degree, as it is of the ſun, and all the primary planets the aforementioned proportion may be expreſſed in this manner, the parallax A D C is to the horizontal parallax A E C, as the ſine of the arch Z F is to the ſine of the arch Z G, or of 90 degrees.

6. The horizontal parallaxes of the celeſtial bodies are inverſely, as their diſtances from the earth's center.

Let A C be the earth, V and S two celeſtial bodies; in the triangle A V S, the ſine of the angle A V S, or of its complement A V C to



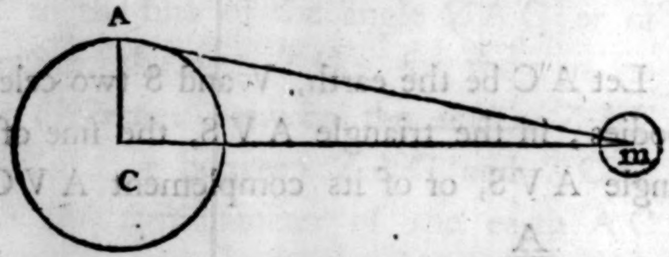
180 degrees, is to that of A S V, as A S is to A V; but in immenſe diſtances from the earth, there is no ſenſible difference between

A S and

A S and C S, and between A V and C V; and when theſe angles are leſs than a minute of a degree, they may be uſed for their ſines, hence the angle A V C, or the parallax of the body at V is to the angle A S C, or the parallax of the body at S, as S C is to V C.

7. The uſe of the parallax of any body is to find its diſtance from the earth's center, and alſo its bulk or magnitude.

Let A C be the earth, M the moon, the angle A M C its horizontal parallax; if this angle was known, the right angle at C, and



the ſemidiameter of the earth being given in the triangle A C M, the line C M would be eaſily found by trigonometry.

Let the angle A M C, or the moon's horizontal parallax be 57′ 18″, the radius A C 4000 miles, then the diſtance of the moon from the earth's center, will come out very near 60 ſemidiameters of the earth, which are 240,000 miles.

It is plain that the diameter of the earth ſubtends an angle of double the horizontal parallax of any body, when the earth is ſeen from it; thus the earth appears to a ſpectator at the moon under an angle of 1° 54′ 36″, but the diameter of the moon as ſeen from the earth, meaſures 31′ 2″; that diameter is to this, as 3,69 to 1; and ſince ſpherical bodies are to one another in a triplicate proportion (or as the cubes) of their diameters; the body of the earth is to that of the moon as 49,4 to unity.

9. *Kepler* firſt found by obſervation, and Sir *Iſaac Newton* hath ſince proved by the laws of gravity, that the ſquares of the periodical times of the primary planets round the ſun, are as the cubes of their mean diſtances from the ſun; but their periodical times are known, and therefore their relative mean diſtances are alſo known: and if the abſolute diſtance of any one of them could be found, the abſolute

diſtances of all the reſt would follow from it; and theſe wonld be obtained if the parallax of the ſun, or any one of the primary planets could be inveſtigated.

10. The magnitude likewiſe of the ſun, and of thoſe planets whoſe apparent diameters have any ſenſible meaſure, would be known from their parallaxes, in the ſame manner as the magnitude of the moon was found.

11. The difference between the real and apparent elevation of any celeſtial body above the horizon is the meaſure of the parallactick angle, and conſequently is the parallax itſelf; and this at any elevation is called the parallax of altitude.

12. If the aſtronomical tables were perfect, and the obſervations ſufficiently accurate for determining the apparent elevation of the ſun, or any one of the primary planets at any time, the end would be anſwered, and the whole doctrine and uſe of parallaxes lie in a very ſmall compaſs.

13. The meridian of any place is generally the beſt azimuth, for taking the apparent elevation of any body above the horizon, in order to find its parallax; becauſe there the real elevation at the center is eaſieſt known from the tables which ſhew either the longitude and latitude of that body from the ecliptic, or its right aſcenſion and declination, from the equator at any given point of time.

Though the horizontal parallax is the largeſt, yet it cannot be commodiouſly taken by obſervation, becauſe the refraction will there raiſe any object as much or more than the parallax depreſſeth it; but that diſtance which is neareſt the horizon, and leaſt affected by refraction, or where it may be eaſieſt allowed for, is the beſt for obſerving the parallax.

When the moon is in her greateſt ſouthern latitude from the ecliptic, and her deſcending node is in the beginning of Libra, ſhe will not be more than ten degrees above the horizon in her paſſage over the meridian of a place whoſe latitude is 51° 20′ north; her true place in the heavens may be very exactly found by the lunar tables, and her elevation above the horizon may be taken within twenty ſeconds of a degree by good aſtronomical inſtruments; if her parallax then at the elevation of ten degrees ſhould appear to be fifty ſix minutes, her horizontal parallax will be 57′ 18″; and if the obſervation be true within twenty ſeconds, her parallax will be aſcertained within the 168th part of it.

15. Hence the moon's parallax may be obtained to a ſufficient degree of exactneſs, by taking her height above the horizon on the meridian, in almoſt any latitude: but the misfortune is, the parallax or diſtance of the ſun or any planet from the earth, hath no connection with the moon's.

The difficulty likewiſe of finding any of theſe parallaxes is enhanced by their ſmallneſs; for the largeſt horizontal parallax of the neareſt planet, even in its perigee, is leſs than fifty ſeconds: and no altitude (if Dr. *Keil's* authority may be relied on) can be taken by the beſt inſtruments within thirteen ſeconds of a degree; or in other words, every arch leſs than thirteen ſeconds is immeaſurable by any quadrant:

drant: Theſe parallaxes therefore cannot be acquired by any ſuch inſtrumental obſervations; and the famous method of *Hipparchus*, for inveſtigating the parallax of the ſun, from the conical ſhadow of the earth, in lunar eclipſes; and alſo the more celebrated dichotomy of *Ariſtarchus* the *Samian*, for determining the ſun's diſtance from the earth, in the quadratures of the moon, however ingenious, are yet defective; becauſe they require obſervations more accurate than inſtruments can give.

16. Aſtronomers indeed have inferred from ſome inſtrumental obſervations, that the parallax of Mars, in its perigee, doth not exceed twenty-five ſeconds of a degree; and conſequently that of the ſun is not more than twelve ſeconds and a half, as the diſtance of Mars at that time is about half the diſtance of the ſun from the earth; this limitation is exceeding probable, though not abſolutely certain; it is alſo ſuppoſed that theſe parallaxes do not fall many ſeconds ſhort of theſe numbers, but how many is very uncertain by reaſon of the imperfect meaſurement of the ſeconds of a degree.

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CHAP. II.

Of the uſe of Time in determining the parallax of any heavenly body.

1. SINCE the parallax of the ſun, or any of the primary planets, is ſo ſmall as to elude the Power of graduated inſtruments, recourſe muſt be had to time; and if a body moves very ſlowly and equably in its orbit, any quantity of its motion may be meaſured with the greateſt preciſion by a good pendulum clock. Let it be ſuppoſed that a celeſtial body moves one minute of a degree in twenty minutes of time; then any part of its orbit may be known to a ſecond, or even to a part of a ſecond of a degree, by knowing the minutes and ſeconds of time that are expended in running it.

2. The velocity of the ſun in the ecliptic, of the moon, and all the planets in their orbits, is according to the time taken up in running them; the mean motion of the moon is at the

rate of thirteen degrees in twenty-four hours, which is almoſt a ſecond of a degree to two ſeconds of time; that of Mercury is nearly a ſecond of a degree to eight ſeconds of time; Venus moves a ſecond of a degree in fifteen ſeconds of time; the ſun in the ecliptic, a ſecond of a degree in twenty-four ſeconds of time; and the motion of the ſuperior planets is ſtill much ſlower.

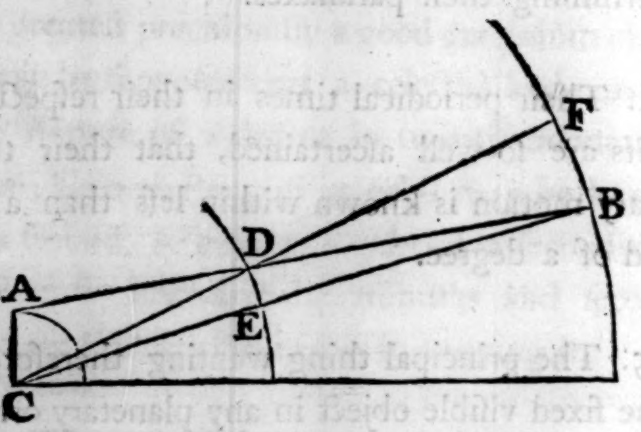
3. The motion of all of them, and even of the moon itſelf, which is far the leaſt regular, is ſufficiently equable in the time required for determining their parallaxes.

4. Their periodical times in their reſpective orbits are ſo well aſcertained, that their true horary motion is known within leſs than a ſecond of a degree.

5. The principal thing wanting therefore is ſome fixed viſible object in any planetary orbit; for if aſtronomical tables will give the time, when a line drawn from the center of the earth through the planet will paſs through that object; and if clocks can give the time, when

the planet is ſeen by an obſerver, upon that object, then the difference of thoſe times turned into degrees, and parts of a degree, according to the motion of that planet in its orbit, will be the parallax of the altitude of that planet above the horizon; provided the planet moves on, or its orbit coincides with an azimuth of the place where it is ſeen.

Let A C be the earth, C the center, A the place of obſervation; let B be the fixed viſible object in the heavens; let D E be the orbit of the planet; when the planet appears upon B



from A, it will be at D; but when it appears at B from C, it will be at E; and D E will be the meaſure of the parallax of altitude of the planet at D.

Draw CF through D, then by ſection the firſt, chapter the firſt; the angle FDB is the parallax of the planet at D ſeen from A, whoſe meaſure is the arch FB, whoſe correſpondent arch is DE, which, being turned into time, is the difference between the times of obſervation at A and C.

N.B. The angles FDB and FCB may be taken as equals, becauſe their difference ABC is ſubtended by the ſemidiameter of the earth, which is but a point, in compariſon of the diſtance of B from it.

6. The path of the ſun, in its diurnal motion, coincides with the eaſtern and weſtern azimuth of the inhabitants on the equator, two days in the year; but that motion is too ſwift to be meaſured by time; on the contrary, that motion is the proper meaſure of time, for the ſun runs fifteen ſeconds of a degree in a ſecond of time; and if it was otherwiſe, there are no objects which the ſun could be referred to in its diurnal motion.

7. But though the ſun hath no ſuch object,

yet is may become itſelf a proper object to the inferior planets, and eſpecially to Venus; which being in conjunction with the ſun, at or near the interior node, will paſs over the diſc of the ſun, aud appear as a large dark ſpot in it.

The ſun indeed hath an apparent motion in the ecliptic, but that may be allowed for, and ſo it may be uſed as a fixed object; the tranſit of Venus over the ſun generally laſts ſome hours, and this planet moving at the rate of a ſecond of a degree in fifteen ſeconds of time, its motion is ſlow enough to have the leaſt part of its orbit meaſured by a good pendulum clock: but the greateſt advantage of all, and which is peculiar to this planet and Mercury, and makes obſervations upon their tranſits ſo ſingularly uſeful for diſcovering the ſun's parallax, is, that their ingreſs upon, and egreſs from the ſun's limb may be aſcertained by an obſerver even to a ſecond of time: I mention this upon the authority of Dr. *Halley*, and becauſe it is a point of great importance with reſpect to the inveſtigation of the parallax of Venus or the ſun, I will quote the Doctor's own words from *Motte*'s Abridgement of the Philoſophical Tranſactions, vol. I. pag. 245.

'About forty years ago, whilſt I was in the
" iſland of St. *Helena*, obſerving the ſtars about
" the ſouth pole, I had an opportunity of obſer-
" ving with the greateſt diligence, Mercury,
" paſſing over the diſc of the ſun, and (which
" ſucceeded better than I could have hoped
" for) I obſerved with the greateſt degree of
" accuracy, by means of a teleſcope twenty-
" four feet long, the very moment when Mer-
" cury entering upon the ſun ſeemed to touch
" its limb within, and alſo the moment, when
" going off, it ſtruck the limb of the ſun's
" diſc, forming an angle of interior contact;
" whence I found the interval of time, during
" which, Mercury then appeared within the
" ſun's diſc, even without an error of one ſe-
" cond of time. For the lucid line intercep-
" ted between the dark limb of the planet and
" the bright limb of the ſun, although exceed-
" ing fine, is ſeen by the eye; and the little
" dent made in the ſun's limb, by Mercury's
" entering the diſc appears to vaniſh in a mo-
" ment: and alſo that made by Mercury, when
" leaving the diſc, ſeems to begin in an inſtant.
" When I perceived this, it immediately came
" into my mind, that the ſun's parallax might

" be accurately determined by ſuch kind of
" obſervations as theſe; provided Mercury was
" nearer to the earth, and had a greater paral-
" lax from the ſun: but the difference of theſe
" parallaxes is ſo little, as always to be leſs
" than the ſolar parallax which we want; and
" therefore Mercury, though frequently to be
" ſeen on the ſun, is not to be looked upon
" as fit for our purpoſe.

" There remains then the Tranſit of Venus
" over the ſun's diſc, whoſe parallax being
" almoſt four times greater than the ſolar pa-
" rallax will cauſe very ſenſible differences be-
" tween the times in which Venus will ſeem
" to be paſſing over the ſun at different parts
" of the earth; and from theſe differences, if
" they be obſerved as they ought, the ſun's
" parallax may be determined even to a ſmall
" part of a ſecond. Nor do we require any
" other inſtruments for this purpoſe than com-
" mon teleſcopes and clocks, only good of
" their kind; and nothing more is needful than
" fidelity, diligence, and a moderate ſkill in
" aſtronomy. For there is no need that the
" latitude of the place ſhould be ſcrupulouſly

" obſerved, nor that the hours themſelves
" ſhould be accurately determined with reſpect
" to the meridian: it is ſufficient that the
" clocks be regulated according to the motions
" of the heavens; if the times be well reck-
" oned from the total ingreſs of Venus into
" the ſun's diſc to the beginning of her egreſs
" from it; that is, when the dark globe of
" Venus firſt begins to touch the bright limb
" of the ſun, which moments, I know by my
" own experience, may be obſerved to within
" a ſecond of time."

8. Hence it is plain, in Dr. *Halley*'s opinion, that Venus is the only planet that furniſhes aſtronomers with any tolerable means for inveſtigating its parallax, or that of the ſun, or of any primary planet.

And if the orbit of Venus coincided with an azimuth of any place for the time of its tranſit over the ſun, there would be no difficulty in finding its parallax, and both the theory and practice of it would be clear, conciſe, and eaſy.

9. But ſince this doth never happen, aſtro-

nomers are obliged to proceed in an analitical or tentative manner, they ſuppoſe an horizontal parallax of Venus from the ſun, and conſequently a parallax of any given altitude as near the truth as they can: this parallax muſt be reſolved into a parallax of longitude, and a parallax of latitude, that the computed times of its ingreſs or egreſs at any given place which this ſuppoſed parallax gives, may be compared with the central times given by aſtronomical tables, or with the computed times of its ingreſs or egreſs at any other given place; and if their difference proves to be the ſame, which the obſerved times of ingreſs or egreſs at the ſame place make with the aſtronomical tables, or with the obſerved times in the other given place, then the parallax was rightly aſſumed; but ſhould their difference not prove to be the ſame, the true parallax will ſoon be found from the difference of their differences.

10. Thus the end deſired may be obtained to a ſufficient degree of exactneſs, provided the obſervations are accurate, but the means to it are operoſe, and the doctrine of parallaxes is thereby rendered ſuch a complicated and intri-

cate ſubject, that an able mathematician, without due care, may ſoon perplex and loſe himſelf.

The purport therefore of the following chapters is to ſet this ſubject in as clear a light as it will admit of; and to explain it in ſuch an intelligible manner, that a perſon moderately ſkilled in the uſe of the globes, and in plane and ſpherical trigonometry, may fully comprehend it.

11. For this end I will begin with a general deſcription of a tranſit of this planet over the ſun, when the node of their orbits is in the middle of the tranſit; and then proceed to an arithmetical and geometrical calculation of the tranſit which was June 6, 1761, and which will be June 3, 1769; and to a compariſon of the central and computed times for moſt of the places where the ingreſs of Venus, its duration upon and egreſs from the ſun in the firſt were ſaid to be obſerved, and where they ought to be obſerved in the laſt.

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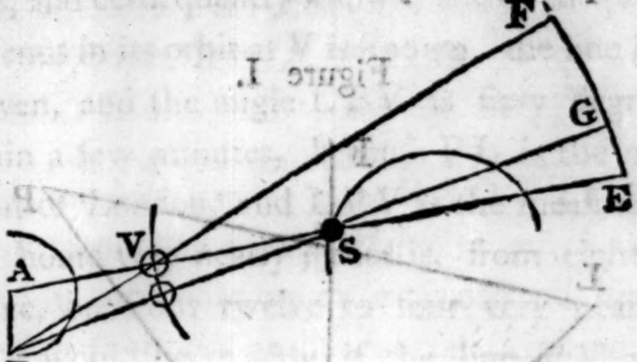
CHAP. III.

BEFORE the general deſcription of the Tranſit of Venus be entered upon, it will be neceſſary to premiſe, that the ſun having a parallax as well as Venus, it will appear depreſſed to an obſerver on the ſurface of the earth, from the place it is ſeen from the center, according to its diſtance, as well as Venus; when therefore this depreſſion of Venus is referred to, compared with, or meaſured from the ſun, it will not be the abſolute depreſſion of Venus, but only the difference of the two parallaxes; and as this relative depreſſion will generally occur, and be made uſe of in the following chapters, it will be diſtinguiſhed by the title of *The parallax of Venus from the ſun.*

Let AB be the earth, V, Venus in its orbit, S a part of the ſun where Venus appears to be; then the arch F E is the parallax of Venus; G E is the parallax of the ſun;



F G is the difference of the two parallaxes, or *the parallax of Venus from the ſun.*

Suppoſe the day, hour, minute, and ſecond of the beginning, middle, and end of a tranſit of Venus over the ſun, to be given by aſtronomical tables.

Let C V D be the orbit of Venus, and let Venus be in its firſt contact at V; let F be the pole of the orbit, and F V will be perpendicular to it; let L be the zenith of the place of obſervation, ſuppoſe it to be London; and let the Time of this contact when ſeen from the center be at eight in the morning as in figure the firſt: or four in the afternoon as in figure the ſecond. Let P be the pole of the equator, and join PV and PL.

Figure I.

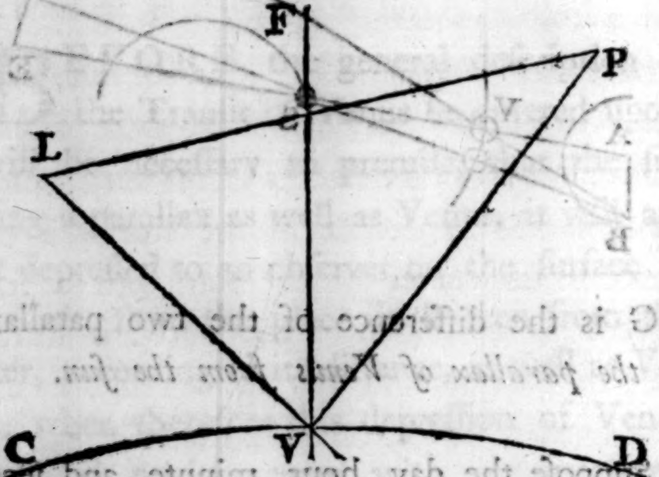
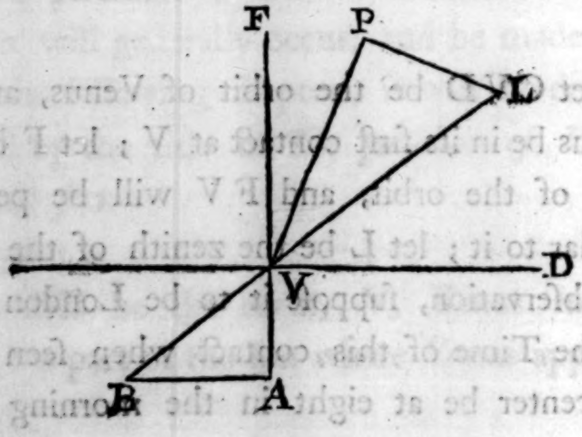


Figure II.

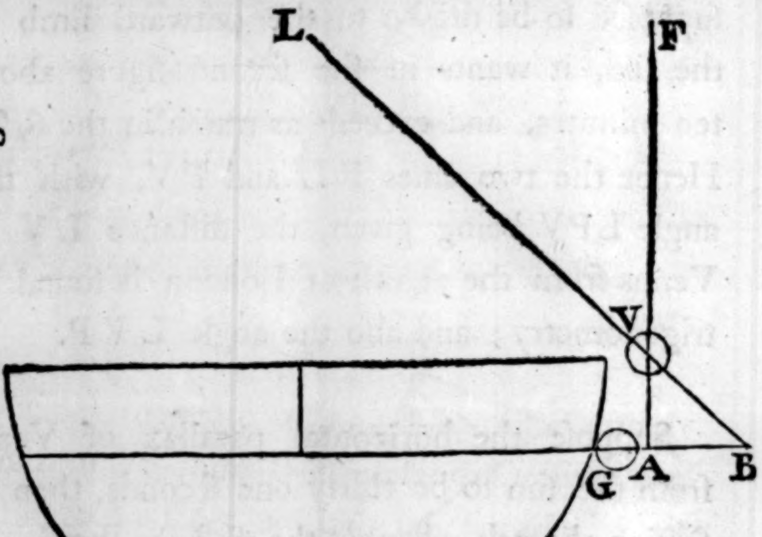


In the triangle L P V the arch L P is the complement of the latitude of London to 90 degrees, and conſequently known, and as the place of Venus in its orbit at V is known, the line PV is given, and the angle L P V is ſixty degrees within a few minutes, becauſe P L is the meridian of London, and L P V is the meaſure of four hours very nearly; that is, from eight to twelve, or from twelve to four very nearly; and it would be exactly if the line P V was drawn to the center of the ſun, but as it is ſuppoſed to be drawn to the outward limb of the ſun, it wants in the ſecond figure about ten minutes, and exceeds as much in the firſt: Hence the two lines P L and P V, with the angle LPV being given, the diſtance L V of Venus from the zenith of London is found by trigonometry; and alſo the angle L V P.

Suppoſe the horizontal parallax of Venus from the ſun to be thirty one ſeconds, then by ſection the 4th, chapter the firſt, as the ſine of 90 degrees, or the radius is to the ſine of 31 ſeconds, ſo is the ſine of the zenith diſtance LV to the ſine of the parallax of that altitude, which is therefore known; let it be B V, and then B is the place where Venus ſeen from London will appear to be, when ſhe is at V as ſeen from the center; and if B V could be meaſured by any inſtrument, and came out the ſame as by computation, the parallax would be found; but as this cannot be, we muſt proceed farther.

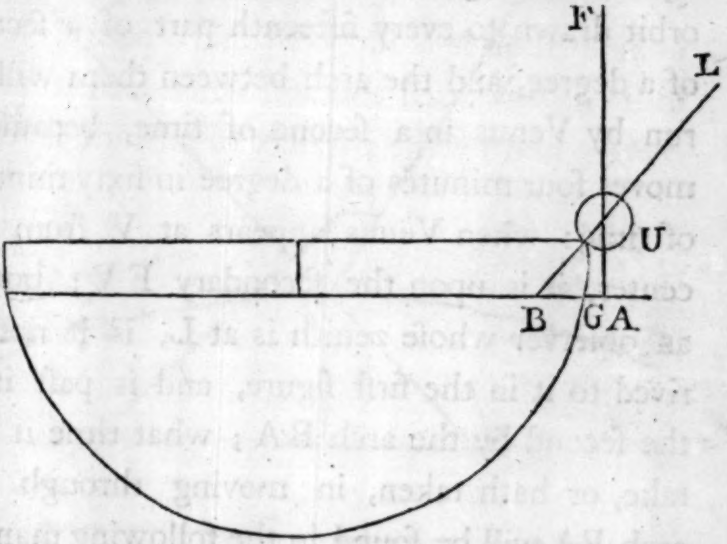
Since then it is ſuppoſed that Venus is at V in its firſt external contact, when ſeen from the center, and by the parallax of altitude is removed to B to an obſerver whoſe zenith is L, ſhe will not appear there to have reached the ſun's limb

Fig. I.



limb by the line G B in the firſt figure, and to have paſſed it by the ſame line in the ſecond figure: it is requiſite therefore to find out how long Venus will be or hath been running this line and appear from London to be at the ſun's limb; and if upon the aſſumption of the parallax, this time will come out the ſame as it appears by obſervation, then the parallax was rightly aſſumed: and in order to make this compariſon, the line G B muſt be meaſured; for which end, let the line VA be drawn from V perpendicular to it, let A B be called the parallax of longitude, and AV the parallax of latitude, and by the help of theſe two lines, G B will be found.

Fig. II.



The arch FV drawn from the pole of the orbit of Venus to the orbit is its ſecondary, and at right angles with it, and ſo are all other lines drawn from that pole to the orbit, and are properly the meaſures of the motion of Venus in its orbit, in the ſame manner as meridian circles are of the diurnal motion of the ſun upon the equator: Imagine theſe ſecondaries of Venus's orbit drawn to every fifteenth part of a ſecond of a degree, and the arch between them will be run by Venus in a ſecond of time, becauſe it moves four minutes of a degree in ſixty minutes of time: when Venus appears at V from the center, it is upon the ſecondary F V; but to an obſerver whoſe zenith is at L, is it not arrived to it in the firſt figure, and is paſt it in the ſecond by the arch B A; what time it will take, or hath taken, in moving through this arch BA will be found in the following manner.

The angle P V D may be eaſily diſcovered, from knowing the angle which the orbit of Venus makes with the ecliptic, and that which the meridian makes with the ecliptic, as will be ſhewn below: but when this angle PVD is thus found, its complement F V P to a right angle is known, and the angle L V P hath already been found, therefore the angle L V F or

Fig. I.

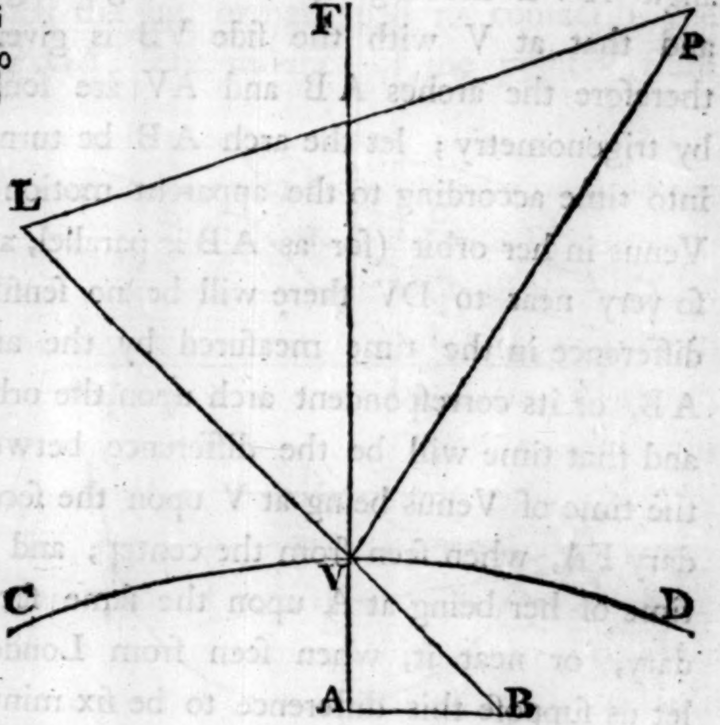
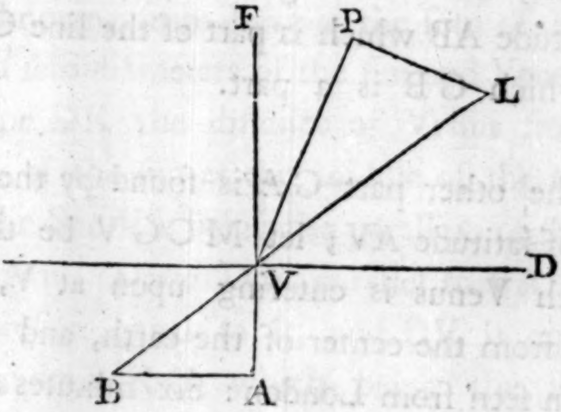
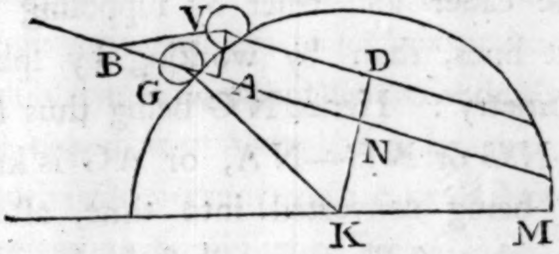


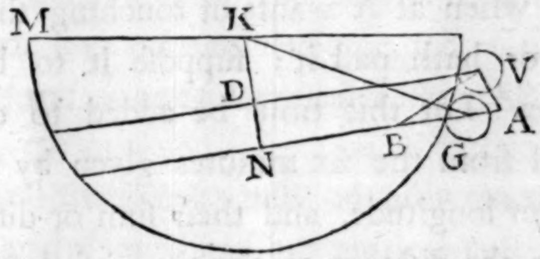
Fig. II.



$LVP \mp FVP$ is alſo known; but L V F is equal to BVA its vertical: Hence in the triangle A V B the angle at A is a right angle, and that at V with the ſide VB is given; therefore the arches A B and AV are found by trigonometry; let the arch A B be turned into time according to the apparent motion of Venus in her orbit (for as A B is parallel, and ſo very near to DV there will be no ſenſible difference in the time meaſured by the arch A B, or its correſpondent arch upon the orbit) and that time will be the difference between the time of Venus being at V upon the ſecondary FA, when ſeen from the center; and the time of her being at A upon the ſame ſecondary, or near it, when ſeen from London; let us ſuppoſe this difference to be ſix minutes of time, and this is given by the parallax of longitude AB which is part of the line G B, or of which G B is a part.

The other part G A is found by the parallax of latitude AV; let M C G V be the ſun, which Venus is entering upon at V, when ſeen from the center of the earth, and is at B, when ſeen from London: Six minutes after or before this time ſhe is at A, as ſeen from thence; but here ſhe hath not yet arrived to touch the ſun, or hath paſt its contact by the line GA: The diſtance of the point V from





D is known, from having the ſum of the apparent ſemidiameters of the ſun and Venus, and the line DK the diſtance of Venus from the center of the ſun at the middle of the tranſit; and the line VA being the parallax of latitude is known. Draw DN parallel to VA, which is therefore equal to it, and DV is equal to NA; draw KG, which being the ſum of the ſame ſemidiameters is known, from whence N G is found by trigonometry from the given parts in the right angle, triangle K N G or by being ſuppoſed equal to $\overline{KG^2 - KN^2}|^{\frac{1}{2}}$ for as theſe arches are ſo very ſmall the operation will be eaſier and truer by ſuppoſing them ſtraight lines, than by working by ſpherical trigonometry: Hence N G being thus found, N A—N G or N G—N A, or A G is known, which being converted into time, after the manner that A B was, will ſhew how much Venus when at A wants of touching the ſun's limb, or hath paſt it; ſuppoſe it to be four minutes: Let this time be added to or ſubtracted from the ſix minutes given by the parallax of longitude, and their ſum or difference will be the diſtance of the time of the central contact of Venus, with the ſun at V from the computed time of the contact at G; if theſe ten or two minutes which came out by computation upon the ſuppoſition of the horizontal parallax of Venus from the ſun being 31 ſeconds, be equal to the diſtance of the ſame time of the central contact from the obſerved contact, then it is evident that the parallax was rightly aſſumed; but if they be not, their exceſs or defect being known, the true parallax will be diſcovered by the next aſſumption and approximation.

This is the general method of inveſtigating the parallax of Venus from the ſun; but there is a variety of particular caſes comprehended in it; the obſerver is here ſuppoſed to be on the north ſide of the orbit of Venus, but he may be on the ſouth ſide; the parallax of longitude is here larger than the parallax of latitude, but it may be leſs; there may be alſo a parallax of longitude and none of latitude, or a parallax latitude and none of longitude.

But all theſe caſes will be fully exemplified in the arithmetical and geometrical calculations of the tranſits of Venus over the ſun, which will be treated of in the ſubſequent chapters.

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CHAP. IV.

THE tranſits of Venus over the ſun are of three kinds,

The firſt is, when the node is before the tranſit.

The ſecond when it is after it.

And the third when it is in the tranſit: This laſt rarely happens; and as it is compounded of the other two, and may be reſolved into them, there will be no occaſion to enter into any particular diſcuſſion of it.

The order of time leads me to begin with the tranſits which are after the node, becauſe one of them was on June 6, 1761; and then I will proceed to the tranſits before the node, and eſpecially to that which will be June the 3d, 1769.

The elements which I ſhall uſe for the calculation of the tranſit at June 6, 1761, are what Mr. *Ferguſon* gives as the moſt correct, and by which he conſtructed his large projection of this tranſit, viz.

The true conjunction of the ſun and Venus was at 50 minutes 32 ſeconds after five in the morning by the time at London.

The ſun's ſemidiameter is 15′ 50″.

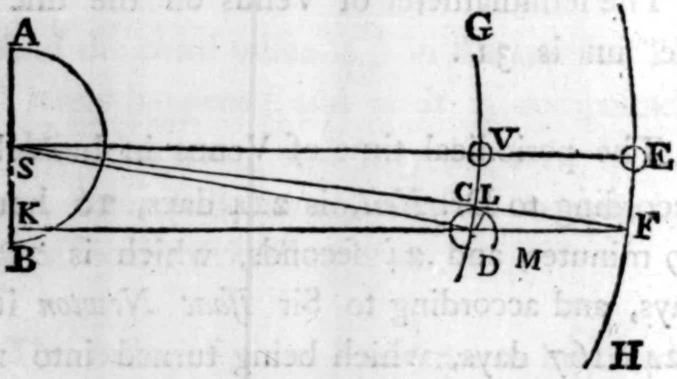
The ſemidiameter of Venus on the diſc of the ſun is 31″.

The periodical time of Venus in her orbit, according to Dr. *Keil,* is 224 days, 16 hours, 49 minutes, and 24 ſeconds, which is 224,7 days, and according to Sir *Iſaac Newton* it is 224,6167 days, which being turned into minutes and divided by 360 degrees or 21,600 minutes will give 14,98 minutes for the firſt, and 14,97 for the laſt, which is very near to 15 minutes of time for one minute of a degree, and conſequently the mean horary motion of Venus is 4′ 0″, 24 of a degree.

The equated horary motion of Venus on the ſixth of June 1761 was 3′ 58″.

The equated horary motion of the earth in the ecliptic at the ſame time was 2′ 23″333. And as they both move the ſame way, their motions muſt be ſubtracted from each other, and their difference will be 1′ 34,″666.

Let ASB be the ſun, VG the orbit of Venus, E H the ecliptic or orbit of the earth, draw SVE where Venus and the earth are ſup-



poſed to be in conjunction; let the earth move in an hour to F, and join SF, and ſuppoſe Venus to move in the ſame time to D, draw S D, and F D K.

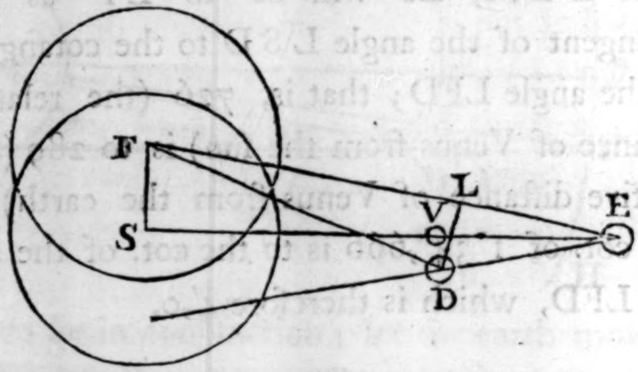
Then C D will repreſent the difference of the two horary motions as ſeen from the ſun, that is, the arch C D, or the angle C S D will appear at S to be 1′ 34″,666. But when the ſame arch is viewed from the earth at F, it will appear under the angle C F D, which will likewiſe be the meaſure of the line S K on the ſun. From D draw DL perpendicular to S F, and as the angle C S D is exceeding ſmall, and the line S L very long, the difference between S L and S C will be ſo little that they may be taken as equals; from the center D and the diſtance D L draw the circle D L M, then as S L and F L are tangents to the ſame circle D L M, LS will be to L F as the cotangent of the angle L S D to the cotangent of the angle LFD; that is, 726 (the relative diſtance of Venus from the ſun) is to 289 (the relative diſtance of Venus from the earth) as the cot. of 1′ 34″,666 is to the cot. of the angle LFD, which is therefore 4′,0.

The ſame likewiſe may be proved in another manner, for in the plane triangle F S D the ſides FS and SD, that is the relative diſtances of the earth and Venus from the ſun, are given with the angle F S D, and from thence the angle C F D will come out four minutes of a degree.

Hence the arch D C in the orbit of Venus or the line S K on the ſun, which repreſents the difference of the two horary motions, and is the apparent geocentrick horary motion of Venus, will be as ſeen from the earth four minutes of a degree.*

---

* Here, by the bye, is a very clear demonſtration of the motion of the earth in the ecliptic; for if the earth was fixed and the ſun moved, the motion of the ſun it is evident muſt be contrary to what the earth's is ſuppoſed to be, to preſerve the ſame aſpects: in the figure therefore annexed, let the ſun and Venus be in conjunction in the line S V E, let Venus move in an hour to D, and the ſun in the ſame time to F; draw D L perpendicular to F E, and the angle E F D will now repreſent the ſum of the two equated horary motions of the ſun, and Venus when ſeen from the ſun; as S F D before did the difference of the two equated horary motions, which will therefore be 6′ 21″,333, and the line L F will be to the line L E as the cot. of the angle 6′ 21″,333 to the cotangent of the angle F E D, which will be 15′ 53′, and is the apparent geocentrick horary motion of Venus upon the ſun. And as the ſun's apparent diameter is but 31′ 40″, Venus would not be two hours in paſſing over the whole diameter of the ſun; but it



From all which it follows that the apparent geocentric motion of Venus, at its interior node, will be very nearly the ſame with its mean motion in its orbit.

If the angle CNV of the node of Venus's orbit with the ecliptic be equal to 3° 23′ 30″ and the diſtance of the ſun CN from the node be 1° 5′ 12″, the right angle at C being given, the arch CV will come out, 3′ 53″, but CV is the meaſure (ſee fig. 2d) of the angle formed at the ſun, by the lines SV drawn from the ſun to Venus, and S C, drawn from the ſun to its apparent place in the ecliptic: Therefore in the plane triangle SCV, ſuppoſing CV to be the ſine of the arch CV, the angle at S and the right angle at C together with the relative diſtance of Venus from the ſun, (viz. SV 726) to the diſtance of Venus from the

it is manifeſt, from the laſt tranſit of Venus over the ſun in 1761, that ſhe was above ſix hours in paſſing over only twenty-four minutes of the ſun's diſc, and would have been eight hours within a few minutes in paſſing over the whole diameter of the ſun, had it been a central tranſit. The fact therefore ſhews that the geocentrick motion of Venus over the ſun muſt be the difference of the two motions of the earth and Venus, and not the ſum of the two motions of the ſun and Venus, and conſequently the earth muſt move in the eccliptic, and not the ſun.

earth at that conjunction, (viz. 289) being given, the line CV will come out 0,8159, and in the triangle CVE; the lines CV and VE with the right angle VCE being known, they

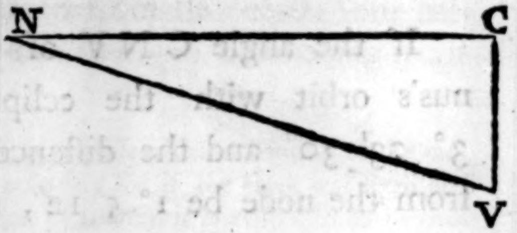


Fig. I.

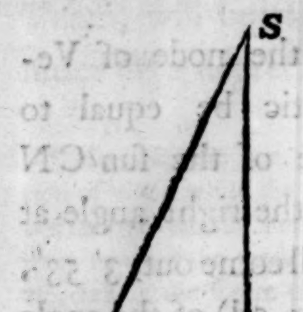
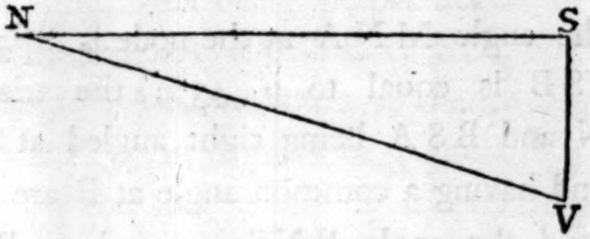


Fig. II.

will give the angle CEV 9′ 43″, which is the geocentric latitude of Venus from the center of the ſun or from the ecliptic at the conjunction.

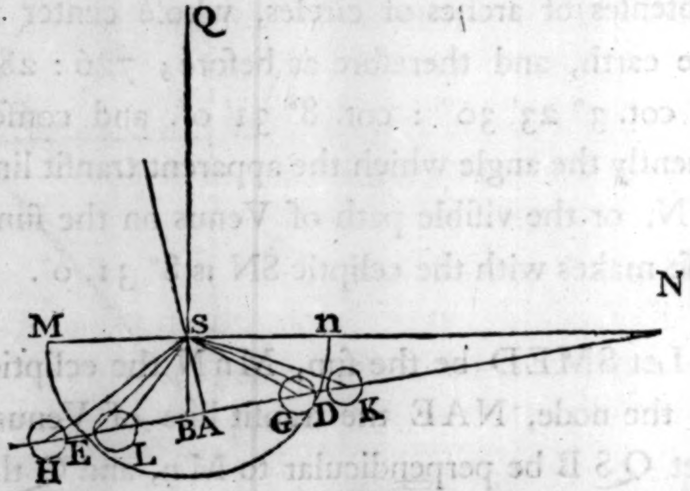
For the ſame reaſon the angle S N V at the node being formed by arches of circles which have the ſun for their center, is as ſeen from the ſun 3° 23′ 30″, but when that angle, or the ſubtenſes of the arches that form it, are viewed from the earth, they will appear to be arches or



ſubtenſes of arches of circles, whoſe center is the earth, and therefore as before; 726 : 289 :: cot. 3° 23′ 30″ : cot. 8° 31′ 0″. and conſequently the angle which the apparent tranſit line V N, or the viſible path of Venus on the ſun's diſc makes with the ecliptic SN is 8° 31′ 0″.

Let SMED be the ſun, MnN the ecliptic, N the node, NAE the tranſit line of Venus: Let QSB be perpendicular to Mn, and Q the pole of the ecliptic; let SA be perpendicular to the tranſit line NAE: Suppoſe the lines SH and SK to be equal to the ſemidiameters of the ſun and Venus, which is 16′ 21″, theſe then ſhew the places of the firſt and laſt external contacts of Venus upon the ſun ; ſuppoſe the lines S L and S G equal to the difference of their ſemidiameters which is 15′ 19″, and they will ſhew where the total ingreſs and beginning of egreſs are.

The angle M N A at the node is 8° 31′ 0″ and S B is equal to 9′ 43″; the triangles B S N and B S A being right angled at S and A, and having a common angle at B are ſimilar, and the angle B N S is equal to B S A, which is 8° 31′ 0″; (theſe triangles are ſo very ſmall that there is no difference whether they are conſidered as plane or ſpherical ; if they be taken as ſpherical, then in the triangle S B N the

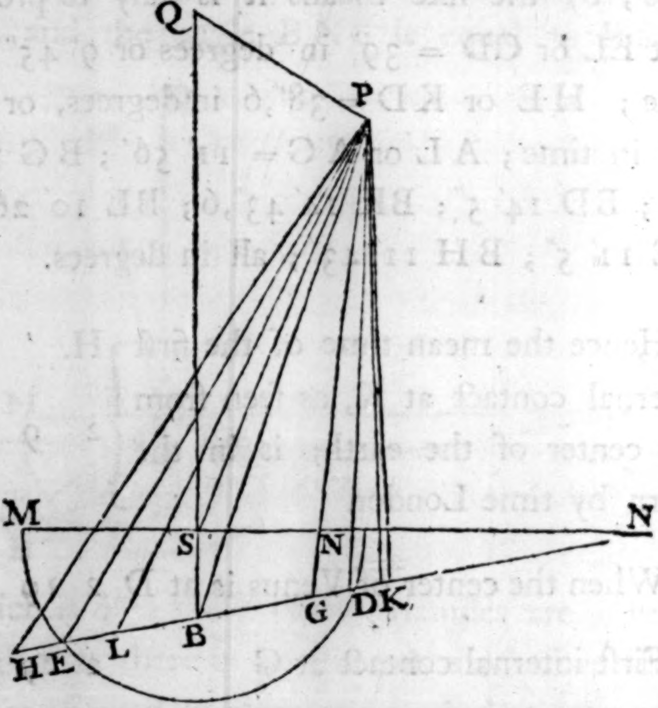


the angles at S and N with the arch S B being given, the angle S B N will be found ; and in the triangle S B A, the angles B and A, with the ſide S B being given, the angle B S A will come out 8° 31′ 0″) and in either caſe SA will be found 9′ 36″ : BA 1′ 30″ in degrees, or 22′ 30″ in time, SA being perpendicular to D E biſects it, and S A with S E or S D, with the right angle S A E are known ; hence E A or A D = 12′ 35″ in degrees, or 3 hours 8′ 45″ in time ; by the like means it is eaſy to prove that EL or GD = 39″ in degrees or 9′ 45″ in time ; H E or K D = 38″,6 in degrees, or 9′ 39″ in time ; A L or A G = 11′ 56′ ; B G 13′ 26″ ; BD 14′ 5″ ; BK 14′ 43″,6 ; BL 10′ 26″ ; B E 11′ 5″ ; B H 11′ 43″ ; all in degrees.

	H.	′	″
Hence the mean time of the firſt external contact at K, as ſeen from the center of the earth, is in the morn by time London	2	9	38
When the center of Venus is at D	2	19	17
Firſt internal contact at G	2	29	2
Middle of the tranſit at A	5	28	2

	H.	′	″
The conjunction at B as given by the tables, and the foundation of all the reſt, is at	5	50	32
Beginning of egreſs, or when the center of Venus is at L	8	27	2
When center of Venus is at E	8	36	47
Ending of the tranſit at H	8	46	26

Let P be the pole of the equator, Q the pole of the ecliptic, N H the tranſit line;



from P draw PQ, PS, PK, PD, PG, PB, PL, PE, PH: In the triangle Q P S, the arch Q S is ninety degrees, being drawn from the ecliptic to its pole ; QP is 23° 30′ 0″ the diſtance of the two poles, and PS is the complement of the ſun's declination at the time of conjunction, which is 67° 17′ 55″: Therefore by trigonometry, the angle QSP comes out ſix degrees, and P S B its complement to two right angles is 174 degrees. Again in the triangle P S B the ſides P S and S B, with the angle P S B are known ; hence the ſide P B is found to be 67° 20′ 51″, and the angle SBP 5° 59′ 37″, the angle P B N = S B N — S B P = 81° 29′ 0″ — 5° 59′ 37″, which is 75° 29′ 23″. In the triangles PBG, PBD, PBK, PBL, PBE, PBH, two ſides and an angle being given, the remaining ſides and angles will be known : Hence,

	°	′	″	Angles	°	′	″	Angles	°	′	″
P K=	67	17	7	P K N=	75	35	23	K P S=	0	6	13
P D	67	17	18	P D N	75	35	5	D P S	0	5	55
P G	67	17	27	P G N	75	34	51	G P S	0	5	41
P B	67	20	51	P B N	75	29	23	B P S	0	0	13
P L	67	23	30	P L N	75	25	3	L P S	0	4	7
P E	67	23	36	P E N	75	24	53	E P S	0	4	17
P H	67	23	48	P H N	75	24	35	H P S	0	4	35

Theſe are preparatory calculations belonging to this tranſit in general, and equally applicable to all places on the ſurface of the earth.

We will now proceed to deſcribe the like general preparatory calculations for the tranſit which will be June the 3d, 1769; becauſe the particular application of theſe generals will more naturally follow each other afterwards.

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CHAP. V.

SOME of the elements for the tranſit June the 3d, 1769, will be the ſame as thoſe uſed in the laſt chapter for the tranſit in 1761.

The true conjuction of Venus with the ſun June the 3d, 1769, will be at 9 hours 59′ 24′ mean time at *Greenwich*, for which I am obliged to the Reverend Mr. *Hornſby*, Savilian Profeſſor of Aſtronomy at *Oxford*.

The ſun's ſemidiameter 15′ 50″, and Venus's ſemidiameter on the diſc of the ſun 31″ as before.

The apparent geocentrick horary motion of Venus on the ſun will be four minutes of a degree, and conſequently Venus will move one minute of a degree in fifteen minutes of time, as before.

The ſun's place at the time of conjunction, by *Dunthrope*'s tables, will be at 2 ſign 13° 26′ 47″.

The place of the node corrected by Mr. *Hornſby's* obſervations on the tranſit in 1761, is at 2 ſign 14° 35′ 34″, as ſeen from the earth: Hence in fig. I. the angle N at the node being given, which is 3° 23′ 30″, the diſtance CN of the ſun from the node which is 1° 8′ 47″, with the right angle at C being given, the line VC will come out 0 4′ 5″, which is

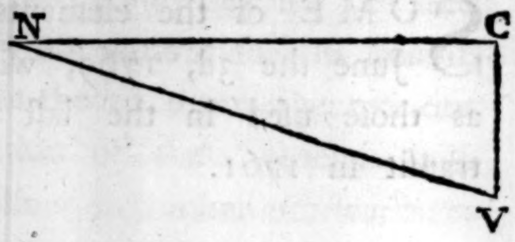


Fig. I.

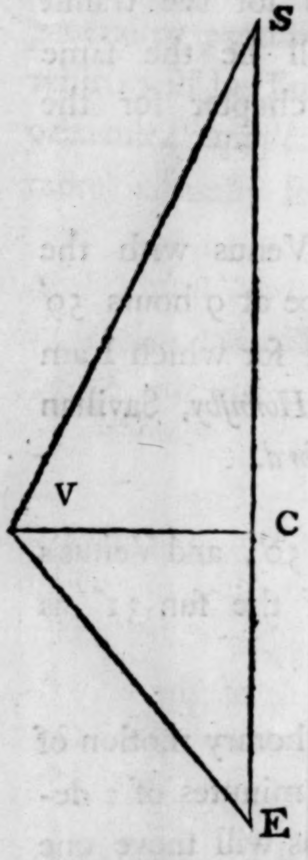


Fig. II.

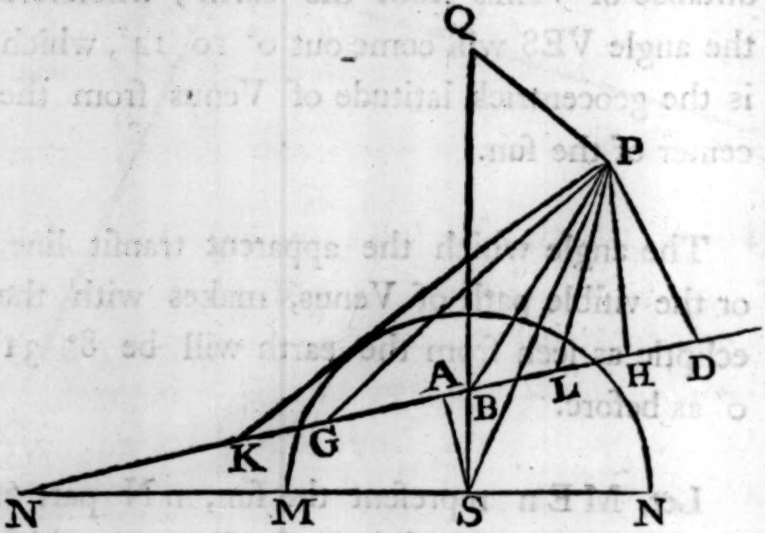
is the meaſure of Venus's latitude from the ecliptic as ſeen from the ſun; and therefore in fig. II. is the meaſure of the angle V S E, but V S is 726 the relative diſtance of Venus from the ſun, and VE is 289, the relative diſtance of Venus from the earth; wherefore the angle VES will come out 0° 10′ 14″, which is the geocentrick latitude of Venus from the center of the ſun.

The angle which the apparent tranſit line, or the viſible path of Venus, makes with the ecliptic as ſeen from the earth will be 8° 31′. 0″ as before.

Let M E n repreſent the ſun, n N part of the ecliptic, D N the tranſit line on which Venus is ſuppoſed to be moving towards the node at N. Let Q be the pole of the ecliptic, P the pole of the equator; draw Q S, Q P, P B, and P S; ſuppoſe S H and S K to be equal to the ſemidiameters of the ſun and Venus, 'and then H and K will ſhew where the firſt and laſt external contacts are: ſuppoſe alſo the lines S L and S G to be equal to the difference of the ſemidiameters of the ſun and Ve-

nus,

nus, and then the points G and L will ſhew where the firſt and laſt internal contacts are. Draw S A perpendicular to G L, and A will be the place of the middle of the tranſit: In the triangle N B S the line B S is 0° 10′ 14″



being the geocentrick latitude of Venus from the center of the ſun; the angle at N is 8° 31′ 0″, and N S B is a right angle; wherefore the angle N B S is found to be 81° 29′ 6″.

Again in the triangle A B S, the angles at A and B, with the line B S being given, the ſide SA will be proved to be 0 10′ 7″,2, and AB 1′ 30″,9 in degrees, or 22′ 43″,5 in time: and in the triangles A S L, A S H, or A S G, ASK, two

A S K, two ſides and a right angle being given, there will be

	′	″		Hours.	′	″	
AG or AL=	11	29,8	in degrees, or	2	52	27	in time.
AK or AH	12	50,4	in degrees, or	3	12	36	in time.
GK or LH	1	20,6	in degrees, or	0	20	9	in time.
BG	13	0,73	in degrees, or	3	15	10,9	in time.
BK	14	21,33	in degrees, or	3	35	19,9	in time.
BL	9	58,9	in degrees, or	2	29	43,5	in time.
BH	11	19,5	in degrees, or	2	49	52,5	in time.

	H.	′	″
Hence the time at firſt external contact at H	7	9	31,5
Of firſt internal contact at L	7	29	40,5
Of the conjunction as given at B	9	59	24
Of the middle of the tranſit at A	10	22	7,5
Of the beginning of egreſs at G	13	14	34,5
Ending of the tranſit at K	13	34	43,5

P S =	67	32	40	complement of the ſun's declination,
QP	23	30	0	diſtance of the two poles,
QS	90	0	0	diſtance of the ecliptic from its pole.

Therefore the angle QSP comes out 7° 3′ 26″; in the triangle BSP the ſides BS and SP, with the angle QSP being given, the angle PBS, or its complement QBP, to two right angles, is found to be 7° 4′ 0″ and PBD =

QBD—QBP = 81° 29′ 6″ — 7° 4′ 0″ = 74° 25′ 6″, and PB = 67° 21′ 40″.

In the triangles PBL, PBH, PBG, PBK, there are two ſides and an angle given, wherefore the following ſides and angles will come out,

	°	′	″	Angles.	°	′	″	Angles.	°	′	″
PK =	67	25	31	PKD =	74	19	22	KPS =	0	16	23
PG	67	25	9	PGD	74	19	55	GPS	0	15	0
PB	67	21	40	PBD	74	25	6	BPS	0	1	26
PL	67	18	59	PLD	74	29	8	LPS	0	9	0
PH	67	18	36	PHD	74	29	53	HPS	0	10	21

CHAP.

CHAP. VI.

Of the addition and ſubtraction of the parallax of longitude and latitude.

1. WHEN Venus is ſeen on the ſun in the eaſtern part of the hemiſphere, or in the forenoon from any point on the ſurface of the earth, and in every kind or part of a tranſit; it is evident that the time which the parallax of longitude AB gives, muſt be added to the central time.

2. But when Venus is ſeen on the ſun in the weſtern part of the hemiſphere, or in the afternoon from any point on the ſurface of the earth, and in every kind or part of a tranſit; the time which the parallax of longitude gives muſt be ſubtracted from the central time.

3. The addition or ſubtraction of the time which the parallax of latitude gives, depends upon the kind of tranſit, the ſituation of the obſerver, and the ingreſs of Venus on the ſun, or its egreſs from it.

4. Let the tranſit be after the node; if the obſerver be on the north ſide of the orbit of Venus, the tranſit line VV will appear to be removed farther from the ecliptic to AA, and conſequently if any ingreſs be obſerved, the time which the parallax of latitude gives muſt be added to the central time.

But ſhould any egreſs be obſerved, the time which the parallax of latitude AV gives, muſt be ſubtracted from it.

5. In the ſame kind of tranſit, if the obſerver be on the ſouth ſide of the orbit of Venus, the tranſit line VV will appear to be removed nearer to the ecliptic as to AA; and therefore the time which the parallax of latitude gives, upon an obſervation of the ingreſs, muſt be ſubtracted from the central time; but ſhould the egreſs be obſerved it muſt be added to it.

6. And conſequently if the ingreſs be obſerved on the north ſide; or the egreſs on the ſouth ſide of the orbit in the morning, the times which the parallaxes of longitude and

latitude give, muſt be both added to the central time.

7. But ſhould the egreſs be obſerved on the north ſide of the orbit, or the ingreſs on the ſouth ſide in the afternoon, then theſe times muſt be together ſubtracted from the central time.

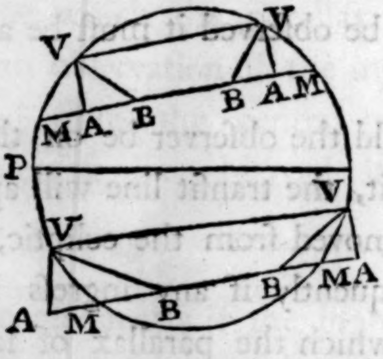
8. Let the tranſit be before the node, then if the obſerver be on the north ſide of the orbit, the tranſit line will appear to be removed nearer to the ecliptic as to AA, and the time which the parallax of latitude VA gives, upon an obſervation of the ingreſs, muſt be ſubtracted from the central time, but ſhould the egreſs be obſerved it muſt be added to it.

9. Should the obſerver be on the ſouth ſide of the orbit, the tranſit line will appear to be farther removed from the ecliptic, as to A A; and conſequently if any ingreſs be obſerved, the time which the parallax of latitude gives muſt be added to the central time; but ſhould the egreſs be obſerved, that time muſt be ſubtracted from it.

10. And conſequently if the egreſs be obſerved on the north ſide of the orbit, or the ingreſs on the ſouth ſide, in the morning, the times which both parallaxes gives muſt be added to the central time.

11. But ſhould the ingreſs be obſerved on the north ſide, or the egreſs on the ſouth ſide in the afternoon, the time which both parallaxes give muſt be ſubtracted from the central time.

In general, of whatever kind the tranſit be, the times which the parallaxes of longitude and latitude give will be of the ſame denomination, that is, to be together added to or ſubtracted from the central time; when the parallactic arch AB, or line of longitude doth not cut the point M of any obſerved ingreſs or



or egreſs: or in other words, when the parallax of longitude A B is ſhorter than the line B M, drawn from the beginning of the parallax of longitude, to the apparent point of ingreſs or egreſs: or again when A falls between M and B.

13. But the parallaxes are of different denominations, that is, one is to be added, and the other ſubtracted; when the reverſe of what hath been ſaid happens, as when the parallactick arch A B cuts the point M, or when M falls between A and B, or B falls between A and M, which laſt may be, though it very rarely happens.

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CHAP. VII.

THE preparations for determining the beſt places, where any ingreſſes of Venus on the ſun, or egreſſes from it, may be obſerved in any given tranſit.

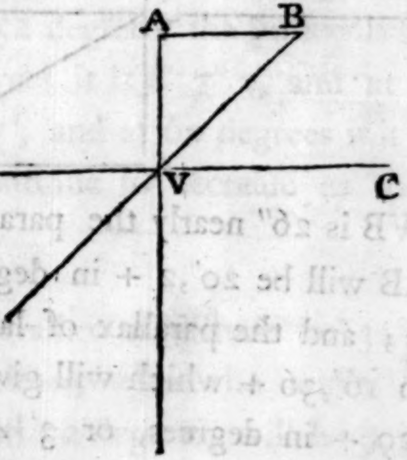
1. Let it be premiſed, that ſuppoſing the lines A B and AV, repreſenting the parallaxes of longitude and latitude equal; the time which the parallax of latitude gives is near one fifth part leſs than that which the parallax of longitude gives; and the reaſon is, becauſe the time, which the parallax of latitude gives, is not meaſured upon the line of that parallax AV, but upon its verſe ſine AM.

2. The places, where there will be the greateſt difference of time, are, where the parallaxes of longitude and latitude are the greateſt, and of the ſame denomination at one place, and contrary to what they are at another; that is, there muſt be the greateſt exceſs of time above the central in one place,

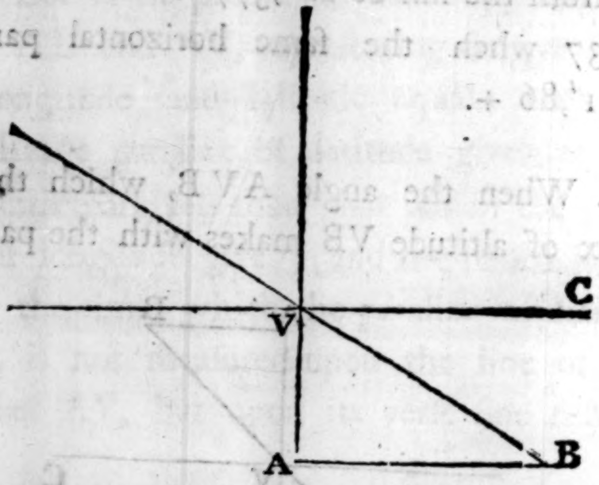
and the greateſt defect from it in the other; and alſo the parallax of altitude muſt be in both places as great as it can conveniently.

3. Let Venus be 80 degrees from the zenith of any place, and let it be ſuppoſed that it is as near the horizon as can be diſtinctly obſerved; then the parallax of altitude will be very nearly 26 ſeconds, if the horizontal parallax of Venus from the ſun be 26″,377 + and it will be 31″,37 when the ſame horizontal parallax is 31″,86 +.

4. When the angle AVB, which the parallax of altitude VB makes with the parallax of latitude AV is 50° 30′ 0″, or between 51 and 52 degrees, the ſum of the times, which the



the parallaxes of longitude and latitude give, will be a maximum: that is, the ſum of thoſe times will decreaſe, if that angle AVB be either leſſer or greater, and the farther it is from 51 and 52 degrees, either in exceſs or defect, the leſſer the ſum of the times will be: for example, let the parallax of altitude VB be as was ſuppoſed either 26″ nearly, or 31″,37, and the angle AVB equal to 51 or 52 degrees.

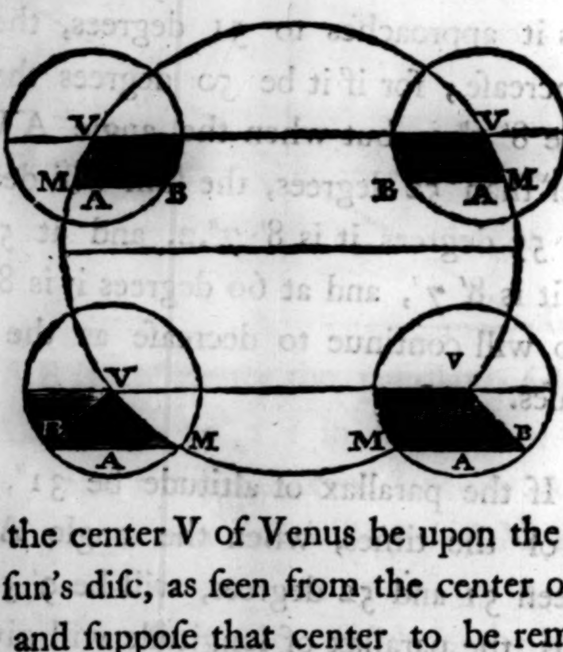


When VB is 26″ nearly the parallax of longitude, AB will be 20″,2 + in degrees, or 5′ 3″ in time; and the parallax of latitude AV is equal to 16″,36 + which will give AM equal to 12″,29 + in degrees, or 3′ 4″,35 in time, whoſe ſum is 8′ 7″,35, which is the largeſt that this parallax of altitude can give, or is larger than any other ſum, when the angle AVB is more or leſs than 51 or 52 degrees: for let the angle AVB be 90 degrees, as the parallax of altitude now coincides with V C, it will be all parallax of longitude; but that parallax being 26″ will give only 6′ 30″ in time; or if the angle A V B be diminiſhed, till VB coincide with the parallax of latitude AV, then it will be all parallax of latitude, and the time will be nearly one fifth part leſs. Again, let the angle AVB be 45 degrees, then the ſum of the times will come out 8′ 4″,3, and as it approaches to 51 degrees, the ſum will increaſe; for if it be 50 degrees the ſum will be 8′ 7″,3, but when the angle A V B is greater than 52 degrees, the ſum will decreaſe, for at 53 degrees it is 8′ 7″,2, and at 55 degrees it is 8′ 7″, and at 60 degrees it is 8′ 1″,8, and ſo will continue to decreaſe as the angle increaſes.

5. If the parallax of altitude be 31″,37 the ſum of the times, when the angle AVB is between 51 and 52 degrees, will be 9′ 51″,3 + that is, the parallax of longitude will give 24″, 38 in degrees, or 6′ 5″,7 in time; and the pa-

rallax of latitude being 19″,74 will give A M 15″,04 in degrees, or 3′ 45″,6 in time; which ſum 9′ 51″,3 + is the maximum for this parallax of altitude.

6. Theſe computations are made upon the ſuppoſition, that the parallax of latitude VA falls without the ſun's diſc; but when it falls within it, the maximum will be at the ſame angle, and the only difference will be, that the ſum of the times, or that maximum may be a few ſeconds larger.



Let the center V of Venus be upon the limb of the ſun's diſc, as ſeen from the center of the earth; and ſuppoſe that center to be removed towards or from the ecliptic C D as at B, when ſeen from the ſurface of the earth; then BM will repreſent the ſum of the times, which the parallaxes of longitude and latitude BA and VA give. The line AV is always perpendicular to the tranſit line VV, but the poſition of the parallax of altitude BV depends upon the ſituation of the obſerver, and when it ſo falls upon AV as to make the angle BVA between 51 and 52 degrees; the line BM will be the longeſt, that can be drawn from any apparent center B of Venus to the ſun's limb M, as was proved by calculation.

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CHAP. VIII.

TO determine the beſt places for obſerving any ingreſſes of Venus upon the ſun, or egreſſes from it, at any given tranſit.

N. B. Though the tranſits of Venus have been all along conſidered only at the deſcending node, or when they are on the north ſide of the equator; yet whatſoever hath been or ſhall be ſaid concerning them will hold good, and may be applied to the tranſits at the aſcending node, or when they are on the ſouth ſide of the equator, with no other difference or alteration than ſuppoſing the two poles to be changed, and always putting the north or ſouth pole or ſide for its contrary, or for the ſouth or north pole or ſide: As it will be therefore unneceſſary to enter into any particulars concerning the tranſits at the aſcending node, we ſhall confine ourſelves, as we have hitherto done, to the tranſits at the deſcending node; and my ſubject leads me to begin with thoſe that are immediately after it.

The firſt thing then to be ſettled in theſe tranſits is the general place where the two parallaxes are of the ſame denomination.

It is evident from the ſixth chapter, that if any ingreſs be obſerved in the morning, and the obſerver be on the north ſide of the orbit of Venus, both the times which the parallaxes of longitude and latitude give, will retard the ingreſs, and muſt be together added to the central time.

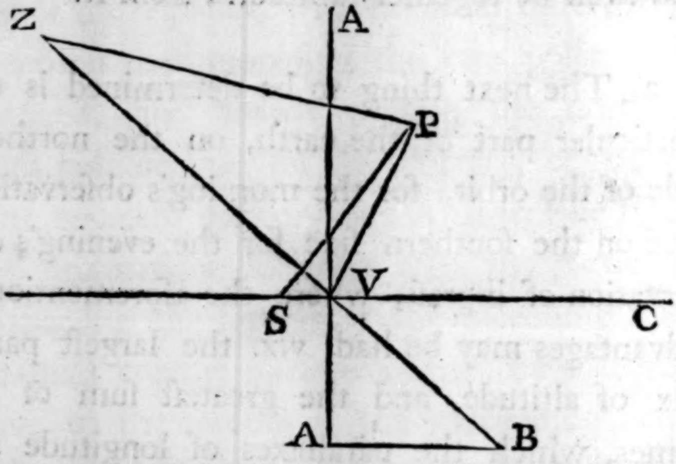
And ſhould the ingreſs in the afternoon be obſerved on the ſouth ſide of the orbit, then thoſe times will accelerate the ingreſs, and muſt be together ſubtracted from it.

2. The next thing to be determined is the particular part of the earth, on the northern ſide of the orbit, for the morning's obſervation, and on the ſouthern ſide for the evening's obſervation of ingreſs, where the aforementioned advantages may be had, viz. the largeſt parallax of altitude, and the greateſt ſum of the times, which the parallaxes of longitude and latitude give.

Theſe will be found in the following manner, for the laſt tranſit of Venus in 1761, and will be very near the ſame in every other tranſit after the node; becauſe there will be very little difference in the angles made by the vertical arch PV, and the orbit of Venus, and in the diſtances PV from the pole, to the orbit in any ſuch tranſits.

3. To determine the place on the northern ſide for the morning's obſervation of the total ingreſs.

Suppoſe the angle BVA or ZVA to be 51 degrees, let Z be the zenith of the place ſought,



and P the north pole; then the angle PVC is equal to 75° 34' 51", (ſee PGN in Gen.

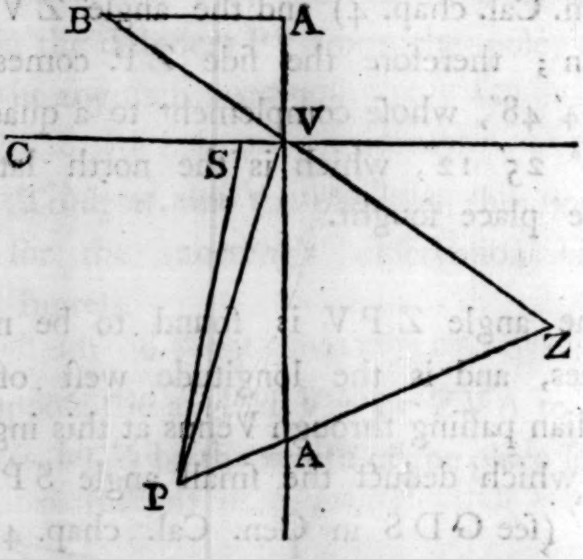
Cal. chap. 4) and A V P its complement to a right angle is 14° 25′ 9″, therefore Z V A + A V P = 65° 25′ 9″ = Z V P. In the triangle Z V P the arch Z V is ſuppoſed to be 80 degrees, and PV = 67° 17′ 27″ (ſee P G in Gen. Cal. chap. 4) and the angle Z V P is known; therefore the ſide Z P comes out 63° 34′ 48″, whoſe complement to a quadrant is 26° 25′ 12″, which is the north latitude of the place ſought.

The angle Z P V is found to be ninety degrees, and is the longitude weſt of the meridian paſſing through Venus at this ingreſs; from which deduct the ſmall angle S P V of 5′ 41′ (ſee G D S in Gen. Cal. chap. 4) and turn them into time, after the rate of fifteen degrees to an hour; and they will make ſix hours within a few ſeconds; and conſequently this ingreſs will be at that place at ſix in the morning nearly.

4. To determine the place on the ſouthern ſide for the evening's obſervation of the total ingreſs.

Let

Let B V A or A V Z be 51 degrees, and P V C will be as before 75° 34′ 51″, and PVA = 14° 25′ 9″, hence PVA + AVZ = 65° 25′ 9″ = P V Z; the ſides P V and Z V are given, that is, P V = 112° 42′ 23″ (ſee Gen. Cal. chap. 4) and V Z is ſuppoſed to be 80 degrees, therefore P Z comes out 71° 53′ 17″, whoſe complement to a quadrant is 18° 6′ 43″ which is the ſouthern latitude of the place ſought.



The angle Z P V is equal to 70° 25′ 56″, and is the longitude eaſt of the meridian paſſing through Venus at this ingreſs, to which add the little angle S P V = 5′ 41″ and turn them into time, after the rate of 15 degrees to an hour, and they will be nearly four hours and forty-two minutes, and therefore the ingreſs will be at that time in the afternoon at that place.

5. The morning's apparent ingreſs on the north ſide of the orbit is retarded by 8′ 7″35 in time, if the parallax of Venus be 26″,377; and 9′ 51″,3 if it be 31″,86 +.

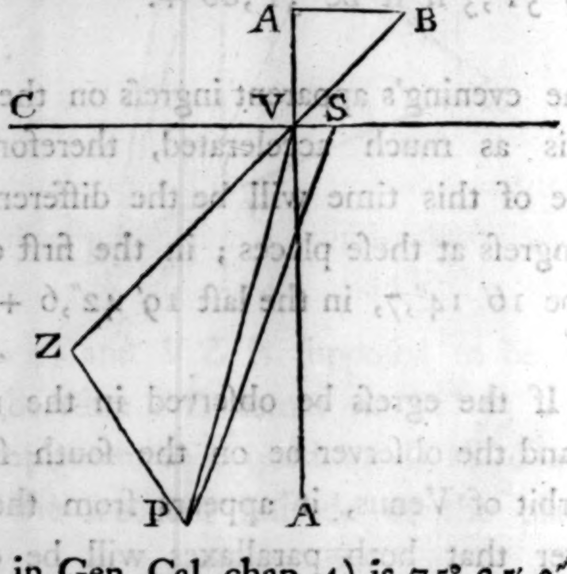
The evening's apparent ingreſs on the ſouth ſide is as much accelerated, therefore the double of this time will be the difference of this ingreſs at theſe places; in the firſt caſe it will be 16′ 14″,7, in the laſt 19′ 42″,6 +.

6. If the egreſs be obſerved in the morning, and the obſerver be on the ſouth ſide of the orbit of Venus, it appears from the ſixth chapter that both parallaxes will be of the ſame denomination, and to be added together to the central time.

And if the egreſs be obſerved in the afternoon from the north ſide of the orbit, then the parallaxes will be of the ſame kind, and to be together ſubtracted from the central time.

7. To determine the place on the ſouth ſide for the beginning of egreſs in the morning.

Let the angle AVB or ZVA be 51 degrees, let Z be the zenith of the place ſought, and P the ſouth pole, then the angle PVC (ſee

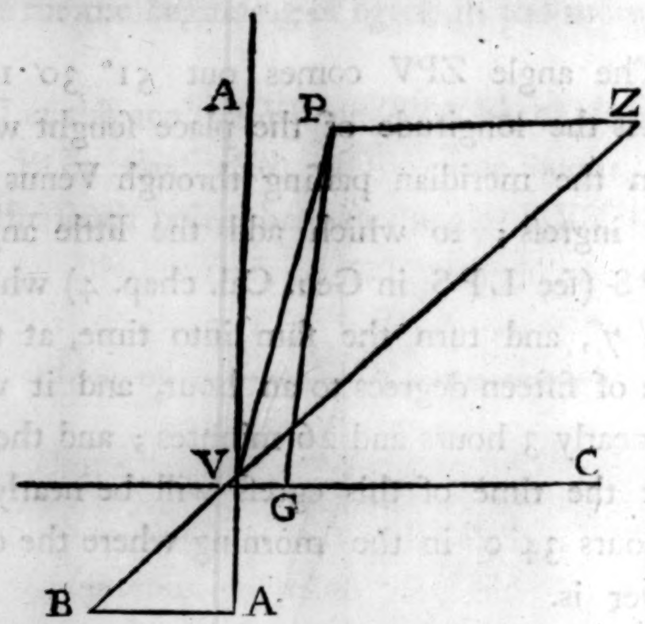


PLN in Gen. Cal. chap. 4) is 75° 25′ 3″, and PVA is 14° 34′ 57″; which ſubtract from ZVA = 51 degrees, and there will remain ZVP = 36° 25′ 3″. ZV is ſuppoſed 80 degrees, and PV is 112° 36′ 30″ (the complement of PL, ſee Gen. Cal. chap. 4, 67° 23′ 30″ to a ſemicircle) hence the ſide ZP comes out 48° 19′ 50″, whoſe complement to 90 degrees is 41° 40′ 10″, and is the ſouthern latitude of the place ſought.

The angle ZPV comes out 51° 30′ 19″, and is the longitude of the place ſought weſt from the meridian paſſing through Venus at this ingreſs; to which add the little angle VPS (ſee LPS in Gen. Cal. chap. 4) which is 4′ 7″, and turn the ſum into time, at the rate of fifteen degrees to an hour, and it will be nearly 3 hours and 26 minutes; and therefore the time of this egreſs will be nearly at 8 hours 34′ 0″ in the morning where the obſerver is.

8. To determine the place on the northern ſide of the orbit, for the evening's obſervation of the beginning of egreſs.

Let the angle BVA or AVZ be 51 degrees, then ZVC is 39 degrees, but PVC is 75° 25′ 3″, (ſee Gen. Cal.) therefore PVZ is 36° 25′ 3″, and the ſide PV is 67° 23′ 30″, (ſee Gen. Cal.) and ZV is 80 degrees by ſuppoſition; hence the arch PZ is found to be 37° 1′ 40″, whoſe complement to a quadrant is the latitude of the place ſought, and is 52° 58′ 20″.



The angle ZPV is found to be either 76° 7′ 52″, or 103° 52′ 8″ its complement to two right angles, for the caſe is ambiguous in the trigonometrical canons; but the particular circumſtances of this caſe require it to be 103° 52′ 8″, from which ſubtract the ſmall angle VPS of 4′ 7″, and turn the reſidue into time, after the rate of fifteen degrees to an hour, and the beginning of this egreſs will be at 6 hours 55′ 30″ nearly; and conſequently the beginning of egreſs will be at that time in the afternoon; and the longitude of the place ſought will be 103° 52′ 8″ eaſt from the meridian paſſing through Venus at that egreſs.

As the morning's apparent egreſs on the ſouth ſide of the orbit, was later than the central egreſs; and the evening's apparent egreſs was ſooner, their difference will be 16′ 14″,7, if the parallax of Venus from the ſun be 26″,377, and 19′ 42″,6, if it be 31″,86 +.

9. From the foregoing ſections of this chapter, it appears, that the latitudes and longitudes of places for obſerving the ingreſs and egreſs of Venus in the morning and evening are as follow:

I. For the morning's obſervation of the ingreſs.

Latitude north 26° 25′ 12″
Longitude weſt 90 0 0 of the meridian
 paſſing through that ingreſs.

II. For the Evening's Ingreſs.

Latitude ſouth 18° 6′ 43″
Longitude eaſt 70 25 56

III. For the morning's obſervation of the egreſs.

Latitude ſouth 41° 40′ 10″
Longitude weſt 51 30 19 of the meridian paſſing through that egreſs.

IV. For the evening's egreſs.

Latitude north 52° 58′ 20″
Longitude eaſt 103 52 8

10. Theſe latitudes and longitudes of the places of obſervation will very well ſerve for the ingreſs and egreſs in any tranſit over the ſun after the node.

The latitudes are general, and the ſame in all tranſits after the node.

But as the longitudes are diſtances from the meridian, paſſing through Venus at any ingreſs or egreſs, they muſt be applied to and meaſured from the place where the ingreſs or egreſs of every particular tranſit after the node is vertical.

Hence if the exact time of an ingreſs or egreſs at any particular tranſit after the node be given, theſe general longitudes will be eaſily reduced to the meridian of any given place, and meaſured from it: Thus for inſtance, let the tranſit be that which was June 6, 1761, let the place be London, to whoſe meridian the aforementioned longitudes are to be applied and reduced: The time of the ingreſs of Venus at that tranſit was at 2 hours 29′ 2″ in the morning of the 6th of June 1761, and the beginning of egreſs was at 8 hours 27′ 2′ in the morning, by London time; theſe times being turned into degrees, at the rate of fifteen degrees to an hour, the firſt will be 37° 15′ 30″, and the other 126° 45′ 30″; it is evident therefore that the places where Venus was vertical at that ingreſs and egreſs, were ſo many degrees deducted from 180 reſpectively in longitude eaſt from the meridian of London, that is, for the ingreſs 142° 44, 30″, and for the egreſs 53° 14′ 30″.

But the longitudes of the beſt places for obſerving the ingreſs or egreſs of any ſuch tranſit being deſcribed as above, they will be eaſily reduced to the meridian of London, and are as follow:

I. For the morning's obſervation of the ingreſs at the tranſit June 6, 1761.

Latitude north	26°	25′	12″	
Longitude eaſt	52	44	30	of London.

II. For the evening's obſervation of the ingreſs.

Latitude ſouth	18°	6′	43″	
Longitude eaſt	213	10	26	of London.
or weſt	146	49	34	of London.

III. For the morning's obſervation of the beginning of egreſs.

Latitude ſouth	41°	40′	10″	
Longitude eaſt	1	44	11	of London.

IV. For the evening's obſervation of the beginning of egreſs.

Latitude north	52°	58′	20″	
Longitude eaſt	157	6	38	of London.

The moſt commodious places for obſerving the total ingreſs nearеſt theſe latitudes and longitudes were

I, The mouth of the River Euphrates, where it empties itſelf into the Perſian Gulph.

II. In the Great Pacifick Ocean, where there is no land, except two or three very ſmall iſlands, called by the names of the Dog Iſle, the Fly Iſland, and Waterland; and theſe are ten degrees too much north.

The moſt commodious places for obſerving the egreſs, and neareſt to theſe longitudes and latitudes were,

III. The extremeſt ſouthern part of the Cape of Good Hope.

IV. The moſt northern part of the Japanick Iſlands, or the northern coaſt of the land of Jeſſo, above the Iſland of Niphonia.

The places where it was ſuppoſed, in the former edition of this book, that the obſervations upon this tranſit were made, are the Iſland of St. Helena, London, Bencoolen in the iſland of Sumatra in the Eaſt Indies, and the Eaſtern Mouth of the River Ganges.

The times which the obſervations at all theſe places ought to give, as found by calculation, differing from the central time either in exceſs or defect, were as here ſet down.

The latitude of St. Helena was ſuppoſed 15° 55′ 0″ ſouth, and its longitude ſix degrees weſt of the meridian of London.

St. HELENA.

The beginning of egreſs.

Par. of Venus from the ſun, 26″,377 or 31″,86

Par. of longitude gave	6′ 0″,4 +	7′ 15″,9 +
Parallax of latitude	1 10,5 +	1 25,5 +
Sum	7 10,9 +	8 41,4 +

Laſt external contact.

Parallax of longitude	5′ 45″,1 +	6′ 57 +
Parallax of latitude	1 21, +	1 36 +
Sum	7 6,1 +	8 33 +

LONDON.

LONDON.

The latitude north ſuppoſed to be 51° 30′ 40″

Beginning of egreſs.

Parallax of longitude	2′ 13,3 +	or	2′ 41″ +
Parallax of latitude	3 45 —		4 30 —
Difference	1 31,7 —		1 49 —

Laſt external contact.

Parallax of longitude	2′ 1 +	2′ 26″,4 +
Parallax of latitude	3 0 —	3 49,5 —
Difference	0 59 —	1 23,1 —

BENCOOLEN.

South latitude 3° 10′ 0″

Longitude eaſt 102 0 0 of the meridian of London.

Firſt external contact.

Parallax of longitude	5′ 4″,3 +	6′ 7″,6 +
Parallax of latitude	0 46,5 —	0 55,5 —
Difference	4 17,8 +	5 12,1 +

Total ingreſs.

Parallax of longitude	4' 43",5	+	5' 42",4	+
Parallax of latitude	0 54	—	1 6	—
Difference	3 49,5	+	4 36,4	+

Beginning of egreſs.

Parallax of longitude	4' 17",1	—	5' 10",6	—
Parallax of latitude	2 28,5	+	3 3	+
Difference	1 48,6	—	2 7,6	—

Laſt external contact.

Parallax of longitude	4' 39",3	—	5' 38",1	—
Parallax of latitude	2 28,5	+	2 48	+
Difference	2 10,8	—	2 50,1	—

Total duration from the firſt to the laſt external contact is ſhortened by

6' 28",6 or 8' 2",2

Total duration from the firſt internal contact to the beginning of egreſs is ſhortened by

5' 38",1 or 6' 44"

G A N G E S.

North latitude 22° 0′ 0″

Longitude eaſt 89 0 0 of the meridian of London.

Firſt external contact.

Parallax of longitude	4′ 47″,5	+	5′ 47″,2	+
Parallax of latitude	1 34,5	+	1 57	+
Sum	6 22	+	7 44,2	+

Firſt internal contact.

Parallax of longitude	4′ 33″,1	+	5′ 30″	+
Parallax of latitude	1 40,5	+	2 3	+
Sum	6 13,6	+	7 33	+

Beginning of egreſs.

Parallax of longitude	3′ 33″,1	—	4′ 17″4,	—
Parallax of latitude	0 27	+	0 33	+
Difference	3 6,1	—	3 44,4	—

Laſt external contact.

Parallax of longitude	3′ 58″,5	—	4′ 48″,1	—
Parallax of latitude	0 30,	+	0 34,5	+
Difference	3 28,5	—	4 13,6	—

Total duration from the firſt to the laſt external contact is ſhortened by 9′ 50″,5 or 11′ 57″,8

Duration from total ingreſs to the beginning of egreſs is ſhortened by 9′ 19″,7 or 11 17,4

Total of parallaxes at the beginning of egreſs.

Helena	7′ 10″,9	+	8′ 41′,4	+
London	1 31,7	—	1 49	—
Bencoolen	1 48,6	—	2 7,6	—
Ganges	3 6,1	—	3 44,4	—

Difference between

Helena and London	8′ 42″,6	or	10′ 30″,4
Helena and Bencoolen	8 59,5	or	10 49
Helena and Ganges	10 17	or	12 25,8

Difference of Duration at Bencoolen and Ganges from firſt to laſt external contact

3′ 21″,9 or 3′ 55″,6

From total ingreſs to the beginning of egreſs.

3′ 41″,6 or 4′ 33″,4

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CHAP. IX.

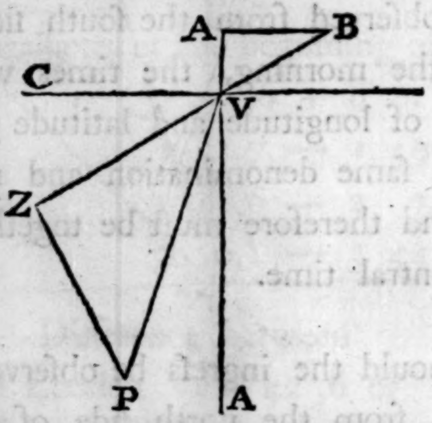
Concerning the tranſits of Venus before the node.

1. IT is evident from the 6th chapter, and the 10th and 11th ſections, that in tranſits of Venus before the node, if the ingreſs be obſerved from the ſouth ſide of the orbit in the morning, the times which the parallaxes of longitude and latitude give, will be of the ſame denomination and retard the ingreſs, and therefore muſt be together added to the central time.

And ſhould the ingreſs be obſerved in the afternoon, from the north ſide of the orbit, then the times which both parallaxes give will accelerate the ingreſs, and muſt be ſubtracted from the central time.

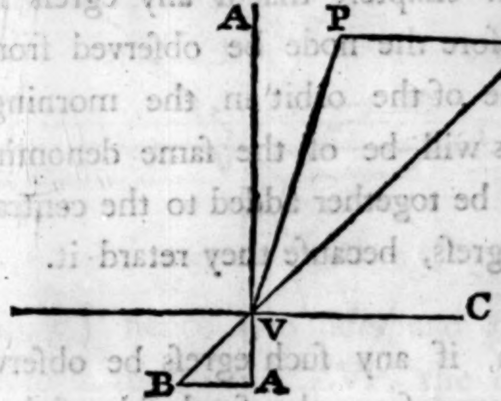
2. To determine the beſt place on the ſouthern ſide, for the morning's obſervation of total ingreſs at the tranſit, June 3, 1769.

Suppoſe the angle BVA or ZVA = 51° 0′ 0″, the angle PVC = 74° 29′ 8″ (ſee PLD in Gen. Cal. chap. 5.) and PVA = 15° 30′ 52″ and PVZ = 35° 29′ 8″, and VZ is ſuppoſed to be 80 degrees: The ſide VP is equal to 112° 41′ 1″ or 67° 18′ 59″ (ſee Gen. Cal. chap. 5.) therefore two ſides and an angle being given in the triangle PVZ, the ſide ZP will come out 47° 42′ 32″, whoſe complement to a quadrant = 42° 17′ 28″, the latitude ſouth of the place ſought: And the angle ZPV = 50° 36′ 27″ longitude weſt from the total ingreſs. But ſhould ZV be ſuppoſed 85 degrees, then ZP will come out 44° 13′ 9″, whoſe complement to 90 degrees is 45° 46′ 51″, and is the latitude ſouth of the place ſought; and the angle ZPV is 56° 1′ 1″ the longitude weſt from the total ingreſs.



3. To determine the place, on the northern ſide of the orbit, for the evening's obſervation of the total ingreſs.

The angle BVA or AVZ = 51 degrees, and PVC as before is 74° 29′ 8″, and AVP = 15° 30′ 52″, the ſide PV is in like manner 67° 18′ 59″, and Z V is ſuppoſed 80 degrees. Wherefore ZP comes out 36° 12′ 48″, whoſe



complement to 90 degrees is 53° 47′ 12″ the latitude north of the place ſought; and the angle ZPV is found to be 104° 36′ 51″, which is the longitude eaſt from the meridian paſſing through the total ingreſs.

If ZV be 85 degrees, the complement of PZ, or the latitude north, will be 51° 26′ 46″, and the angle ZPV or the longitude eaſt will be 111° 53′ 47″.

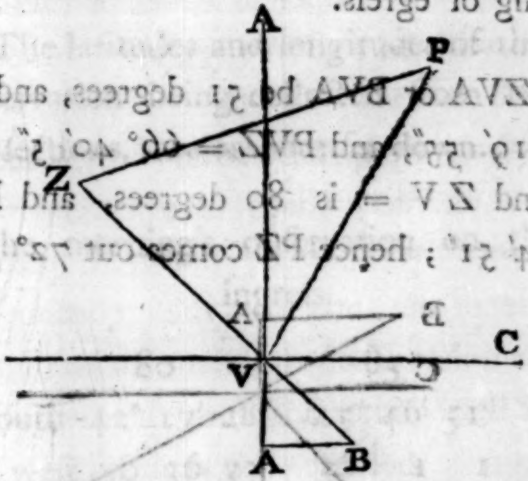
If ZV be ſuppoſed 90 degrees, the complement of PZ, or the latitude north, will be 48° 42′ 9″; and the angle ZPV, or the longitude eaſt, will be 118° 18′ 34″.

4. We learn from the 10th and 11th ſections of the 6th chapter, that if any egreſs in any tranſit before the node be obſerved from the north ſide of the orbit in the morning, the parallaxes will be of the ſame denomination, and muſt be together added to the central time of that egreſs, becauſe they retard it.

Again, if any ſuch egreſs be obſerved in the afternoon from the ſouth ſide of the orbit, then it will be accelerated, and the times which the parallaxes give, muſt be together ſubtracted from the central time.

5. To determine the beſt place, on the northern ſide, for the morning's obſervation of the beginning of egreſs.

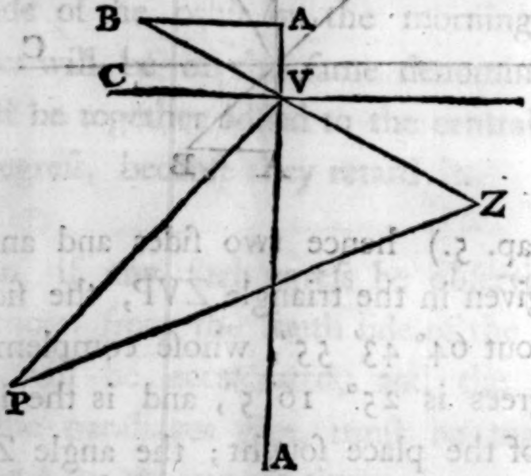
Let ZVA be 51 degrees or AVB, and as PVC is 74° 19′ 55″ (ſee Gen. Cal. in chap. 5.) the angle AVP = 15° 40′ 5″; hence ZVP = 66° 40′ 5″, the arch ZV is ſuppoſed to be 80 degrees, and PV = 67° 25′ 9″ (ſee Gen. Cal. chap. 5.) hence two ſides and an angle being given in the triangle ZVP, the ſide ZP comes out 64° 43′ 55″, whoſe complement to 90 degrees is 25° 16′ 5″, and is the latitude north of the place ſought; the angle ZPV is found to be 89° 25′ 0″, which is the longitude weſt from that egreſs.



Should ZV be 85 degrees, then ZP will come out 66° 33′ 34″ and the latitude north is 23° 26′ 26″; and the angle ZPV, or the longitude weſt is 94° 25′ 48″.

6. To determine the beſt place, on the ſouthern ſide, for the evening's obſervation of the beginning of egreſs.

Let ZVA or BVA be 51 degrees, and PVC is 74° 19′ 55″, and PVZ = 66° 40′ 5″ as before, and ZV = is 80 degrees, and PV = 112° 34′ 51″; hence PZ comes out 72° 56′ 2″, whoſe complement 17° 3′ 58″ is the latitude ſouth of the place ſought; and the angle ZPV is 71° 4′ 20″, and the longitude eaſt from the beginning of egreſs.



Should the zenith diſtance ZV be 85 degrees, then ZP comes out 70° 40′ 51″, and its complement 19° 19′ 9″ is the latitude ſouth of the place ſought; and the angle ZPV is 75° 43′ 2″ and is the longitude eaſt from that egreſs.

7. The latitudes and longitudes of the places of obſervation being collected from the foregoing ſections, are as here ſet down.

For the morning's obſervation on the total ingreſs.

	80	85	90
Lat. ſouth	42° 17′ 28″	45° 46′ 51″	
Long. weſt	50 36 27	56 1 1	

For the Evening's obſervation on the total ingreſs.

Latitude north,	53° 47′ 12″	51° 26′ 46″	48° 42′ 9″
Longitude eaſt,	104 36 51	111 53 47	118 18 34

For the morning's obſervation on the beginning of egreſs.

Latitude north	25° 16′ 5″	23° 26′ 26″
Longitude weſt	89 25 0	94 25 48

For the evening's obſervation on the beginning of egreſs.

Latitude ſouth	17° 3′ 58″	19° 19′ 9″
Longitude eaſt	71 4 20	75 43 2

8. As there are but few minutes between the firſt external contact and the total ingreſs, the ſame place will do very well for the obſervation of both; and for the ſame reaſon, the beginning of egreſs and the laſt external contact may be obſerved from the ſame place.

Though the aforementioned latitudes and longitudes are determined by the angles PLD and PGD, and the arches PL and PG (ſee Gen. Cal. chap. 5) and adapted to the tranſit June 3, 1769, yet as there is very little difference in thoſe angles and arches in any other tranſit before the node, the ſame latitudes and longitudes will give the beſt places of obſervation for every tranſit before the node.

Theſe latitudes and longitudes then are general, and the latitudes always continue ſo, but the longitudes in every particular tranſit before the node muſt be applied to the place where the ingreſs or egreſs of that tranſit is vertical, or to the meridian that paſſes through that ingreſs or egreſs; and when that place or meridian is known, the places of obſervation which are determined by the general latitudes and longitudes may be eaſily referred to and deſcribed, by the latitudes and particular longitudes from any known or primary meridian. Thus for inſtance to apply theſe general latitudes and longitudes to the meridian of Greenwich, for the tranſit in June 3, 1769; the firſt internal contact at that tranſit will be at 7 hours 29′ 40″,5 in the afternoon, which being turned into degrees, at the rate of fifteen degrees to an hour, will ſhew the longitude of Greenwich (if the little angle L P S which is 9 minutes be ſubtracted from it) to be 112° 16′ 7″,5 eaſt of that contact.

In like manner the beginning of egreſs at that tranſit will be at 13 hours 14′ 34″,5 afternoon, which being turned into degrees, at the ſame rate as before, and adding to it the little angle GPS which is fifteen minutes, (ſee Gen. Cal. chap. 5) will ſhew the longitude of Greenwich to be 198° 53′ 37″5, eaſt from that egreſs.

9. Hence the beſt places for obſerving the ingreſs and egreſs at this particular tranſit in 1769, may be deſcribed by their general latitudes and particular longitudes, from the meridian of Greenwich; which will be as follow,

Morning's obſervation of the total ingreſs.

	80 Degrees	85	90
Lat. ſouth	42° 17′ 28″	45° 46′	51″
Long. weſt	162 52 34,5	168 17	8,5

of Greenwich.

Evening's obſervation of total ingreſs.

Lat. north	53° 47′ 12″	51° 26′ 46″	48° 42′ 9″
Long. weſt	7 39 16,5	0 22 20,5	6 2 26,5
		weſt of Greenwich.	eaſt of Green.

Morning's obſervation of the beginning of egreſs.

Lat. north	25° 16′ 5″	23° 26′ 26″	
Long. weſt	288 18 37,5	293 19 25,5	of Greenwich.
or eaſt	71 41 22,5	66 40 34,5	of Greenwich.

Evening's obſervation of the beginning of egreſs.

Lat. ſouth	17° 3′ 58″	19° 19′ 9″	
Long. weſt	127 49 17,5	123 10 35,5	of Greenwich.

N. B. The

N. B. The places given for obſerving this tranſit were wrong in the firſt edition of this book, the miſtake was owing to the ſuppoſition of this tranſit being after the node, and ſimilar to that in 1761, and calculating the places from this falſe ſuppoſition.

10. If the parallax of the ſun be either 8 1-half, or 10 1-half, or 12 1-half, then in the places determined by the aforementioned longitudes and latitudes, for obſerving the total ingreſs, the parallax of longitude will give in time, at the zenith diſtance of 80 degrees from that ingreſs, either

8″ 1-h. ſun's par. 10″ 1-h. 12″ 1-h.

3′ 42″,1 or 5′ 2″,7 or 6′ 5″,7 in time.

Parallax of latitude gives,

2 34,5 or 3 30,0 or 4 12,0 in time.

6 16,6 8 32,7 10 17,7 ſum of the times.

But ſhould the zenith diſtance from Venus at this ingreſs be 85 degrees, then

	8″ 1-h.	10′ 1-h.	12″ 1-h.
Par. of long. gives	3′ 44′,7	or 5′ 6″2	or 6′ 9″9
Par. of lat.	2 36	or 3 31,5	or 4 15
Sum of both times	6 20,7	or 8 37,7	or 10 24,9

Should the zenith diſtance be ninety degrees, then,

Par. of lon. gives	3′ 45″,6	or 5′ 7″,5	or 6′ 11″,4
Par. of lat.	2 36	or 3 31,5	or 4 28,5
Sum of both times	6 21,6	or 8 39	or 10 39,9

11. As both morning's and evening's obſervations of this ingreſs give the ſame times, and as the times of one muſt be added to the central time of this ingreſs, and the times of the other muſt be ſubtracted from it, the difference between them will be double of thoſe times; that is, the difference of the morning's obſervation of total ingreſs from the evening's obſervation will be at 80° diſtance from that ingreſs

	8 1-h.	10 1-h.	12 1-h.
either	12′ 33″,2	or 17′ 05″,4	or 20′ 35″,4
at 85° diſtance	12 41,4	or 17 15,4	or 20 49,8
at 90° diſtance	12 43,2	or 17 18	or 21 19,8

The difference of the morning's obſervation of the beginning of egreſs, from the evening's obſervation of the ſame egreſs, at the places pointed out by the aforementioned longitudes and latitudes, will be near the ſame with the difference between the total ingreſſes, as deſcribed above.

12. The places on the earth which nearly correſpond to the latitudes and longitudes from the meridian of Greenwich, for obſerving the ingreſs and egreſs of Venus on the ſun in the the tranſit, June 3, 1769, are

For the morning's obſervation of total ingreſs,

In the ſouthern part of the great pacifick ocean; but there is no land either at eighty or eighty-five degrees zenith diſtance from this ingreſs, there are indeed a few ſmall iſlands near the equator, called the Dog Iſland, Fly Iſland, Waterland, &c.

Theſe places may have pretty near the true longitudes, but they are 40 degrees diſtant from the true latitudes.

The places for the evening's obſervation of total ingreſs are

Firſt, for the zenith diſtance of 80 degrees from Venus at that ingreſs, the City of Dublin, the metropolis of the kingdom of Ireland; for it nearly correſponds with the given longitudes and latitudes for that evening's obſervation.

Secondly, for the zenith diſtance of eighty-five degrees from Venus, Greenwich is the beſt place.

Thirdly, for the zenith diſtance of ninety degrees from Venus; Paris, or ſome few leagues to the eaſt of that city, becauſe it is not above three or four degrees eaſt of Greenwich.

This ingreſs may be obſerved likewiſe at any part of the Azore Iſlands, at Newfoundland, or Quebec; or any place in North America, from Cape Breton to Georgia.

The places for the morning's obſervation of the beginning of egreſs, which anſwer to the proper longitudes and latitudes, lie in the inland parts of India and China; ſo that any place north of the River Ganges, or any part of the ſea coaſt of China or Nankin, the iſland of Formoſa, or any of the Japannic Iſlands, will not be incommodious places at either zenith diſtance from Venus for the obſervation of this egreſs.

The places for the evening's obſervation of the beginning of egreſs, at either zenith diſtance from Venus, lie in the great ſouth ſea, where there is no land near the given longitudes and latitudes; the extremeſt ſouthern part of the Peninſula of California anſwers pretty near to the given longitudes, but it hath twenty-five degrees north latitude, inſtead of ſeventeen or eighteen ſouth latitude; and therefore it is two or three degrees north of the orbit of Venus, which will make the parallaxes of longitude and latitude contrary to one another, and for that reaſon it will give much leſs time than the proper places would give, but yet it is the beſt place that can be moſt eaſily come at; unleſs there be any iſlands that lie ſouth eaſt of it, and therefore we ſhall make uſe of it for our calculation of the times in its due place.

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CHAP. X.

THE calculation of the times which the parallaxes of longitude and latitude give in particular places.

WATERLAND.

For the firſt external contact.

This iſland is in the great pacifick ocean, and according to our beſt maps is ſituated near 152 degrees longitude weſt of the meridian of Greenwich, and five degrees ſouth latitude, which give a zenith diſtance from Venus at the firſt external contact of the ſun 51° 46′ 55″, the angle which the parallax of altitude makes with the parallax of latitude or AVB will be 78° 47′ 42″; and if the ſun's parallax be ſuppoſed either 8 1-half, 10 1-half, or 12 1-half, then the times which the parallax of longitude at this contact gives are

3′ 43″,6 or 5′ 4″,7 or 6′ 8″,2

The times which the parallax of latitude give are,

$$\frac{0'\ 33'',2}{4\ \ 16,8} \text{ or } \frac{0'\ 46'',5}{5\ \ 51,2} \text{ or } \frac{0'\ 54''}{7\ \ 2,2}$$

ſum of the times.

If this contact ſeen from the center be by Waterland time at $9^h\ 1'\ 31'',5$ in the morning, then the time of this contact ſeen from Waterland will be, by adding the above times to the central, at

$9^h\ 5'\ 48'',3$ or $9^h\ 7'\ 22'',7$ or $9^h\ 8'\ 33'',7$

W A T E R L A N D.

For the total ingreſs.

The zenith diſtance from Venus at this ingreſs will be 47° 40′ 48″, and the angle AVB will be 74° 57′ 51″.

Times of par. of long.	3′ 27″,3	or 4′ 42″,4	or 5′ 41″,2
Parallax of latitude,	0 51	or 1 9	or 1 22,5
Sum of both	4 18,3	or 5 51,4	or 7 3,7

If this contact at the center by Waterland time be at $9^h\ 21'\ 40''5$, then the times of this contact at Waterland will be $9^h\ 25'\ 58''8$, or $9^h\ 27'\ 31'',9$, or $9^h\ 28'\ 44'',2$.

CALIFORNIA.

For the firſt external contact.

The ſouth part of this long Peninſula is about 120 degrees longitude weſt of Greenwich, and 25 degrees north latitude; the zenith diſtance from Venus will be $11^\circ\ 55'\ 7''$, and the angle AVB is $60^\circ\ 40'\ 53''$.

Times of par. of long.	$0'\ 54'',2$	or	$1'\ 11,''2$	or	$1'\ 26''$	+
Parallax of latitude	0 24	or	0 31,5	or	0 39	—
Difference	0 30,2	or	0 39,7	or	0 47	+

If this contact at the center by California time be at $11^h\ 9'\ 31'',5$ in the morning, this contact at California will be at $11^h\ 10''\ 1'',7$ or $11^h\ 10'\ 11'',2$, or $11^h\ 10'\ 18'',5$.

CALIFORNIA.

For the total ingreſs.

The zenith diſtance = $7^\circ\ 26'\ 14''$, the angle AVB = $54^\circ\ 51'\ 46''$.

Times

Times of par. of long.	0′ 30″,7	or	0′ 41″,8	or	0′ 50″,5	+
Parallax of latitude,	0 18	or	0 23,5	or	0 30,5	—
Difference	0 12,7	or	0 18,3	or	0 20	+

If this contact by California time be at the center at $11^{h}\ 29'\ 40'',5$ in the morning, at California it will be at $11^{h}\ 29'\ 53'',2$, or $11^{h}\ 29'\ 58'',8$ or $11^{h}\ 30'\ 0'',5$.

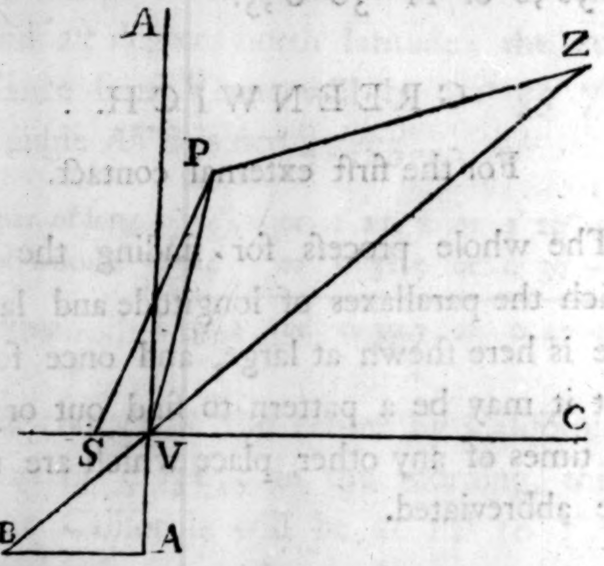
GREENWICH.

For the firſt external contact.

The whole proceſs for finding the times which the parallaxes of longitude and latitude give is here ſhewn at large, and once for all, that it may be a pattern to find out or prove the times of any other place which are therefore abbreviated.

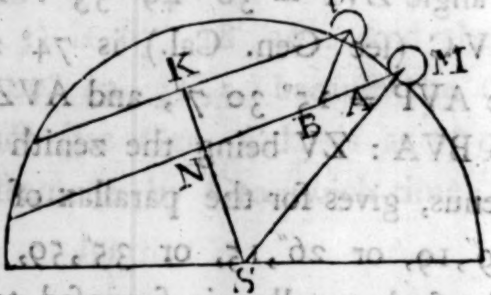
Let SVC be the orbit of Venus, AA an arch drawn from the pole of Venus's orbit, and perpendicular to the orbit; let P be the pole of the equator, and join PV; ſuppoſe Venus at this contact to be at V, moving towards S; let Z be the zenith of Greenwich, and draw ZP and ZV; then in the triangle ZPV, the

ZPV, the arch PZ is the complement of the latitude of Greenwich to 90 degrees, which is 38° 31′ 30″: And the ſide PV (ſee Gen. Cal. chap 5) is 67° 18′ 36″: And the angle ZPV is 107° 12′ 31″,5, becauſe the time of this contact (ſee Gen. Cal.) is at 7° 9′ 31″,5 in the afternoon by Greenwich time; which



being turned into degrees, at the rate of 15 degrees to an hour, make 107° 22′ 52″,5, which is the angle ZPS; if the line PS be ſuppoſed to be drawn through the center of the ſun, but the angle V P S (ſee HPS in Gen. Cal. chap. 5) is 0° 10′ 21″, which ſubtract from ZPS, and there will remain ZPV = 107° 12′ 31″,5: The two ſides therefore in the triangle ZPV being given, with the angle ZPV, the ſide ZV will come out 82° 31′ 25″, and the angle ZVP = 36° 49′ 53″: But the angle PVC (ſee Gen. Cal.) is 74° 29′ 53″, therefore AVP = 15° 30′ 7′, and AVZ = 52° 20′ 0″ = BVA: ZV being the zenith diſtance from Venus, gives for the parallax of altitude either 19″,19, or 26″,15, or 35″,59, according as the ſun's parallax is ſuppoſed to be at 8 1-half, 10 1-half, and 12 1-half; produce ZV to B, ſo that VB be equal to the parallax of altitude, and draw BA parallel to VC, and join VA. As V is the place of Venus at this contact, when ſeen from the center of the earth, B will be the place where Venus appears to be at the ſame time when ſeen from the place where Z is the zenith, and therefore BA will be the parallax of longitude, and VA the parallax of latitude. Now in the triangle BVA the angle at A is a right angle; the angle BVA is known, and the ſide BV is alſo known; from whence BA the parallax of longitude is found to be 15″,19, or 20″,7, or 25″, which give in time either 3′ 47″,8, or 5′ 10″,5, or 6′ 15″. The parallax of latitude = VA comes out 11″,72, or 15″,98, or 19″,3 but VA is equal (ſee fig. 2) to KN, and KS =

Fig. II.



0° 10′ 14″ (ſee Gen. Cal.) therefore SN = 10′ 2″,2, or 9′ 58″,01, or 9′ 54″,7, and SM being the ſum of the ſemidiameters of the ſun and Venus, is 16′ 21″, hence NM is known; and KV or NA = 11′ 19″,5, (ſee BH in Gen. Cal.) therefore NM — NA being equal to AM is either 9″,2, or 12″,5, or 14″,9, which give in time 2′ 18″, or 3′ 7″,5, or 3′ 43″,5. But the times which BA and AM give are the times that the external contact at the center exceeds this contact at Greenwich, which are either 6′ 5″,8 or 8′ 18″, or 9′ 58″,5: The time of this contact at the center will be by Greenwich time at 7^h 9′ 31″,5 (ſee Gen. Cal.) and therefore if we ſubtract the aforementioned times, we ſhall have the time of this contact at Greenwich; which will be 7^h 3′ 25″,7, or 7^h 1′ 13″,5, or 6^h 59′ 33″.

N. B. In

N. B. In all places where this contact may be obſerved the arches AV and PV, and the orbit of Venus VC, are the ſame, or have the ſame ſituation in reſpect to one another; but the arch ZV hath different poſitions, it is here north eaſt in reſpect to PV and the orbit of Venus, but it may be north weſt, or ſouth eaſt, or ſouth weſt, in all which caſes, the figure muſt be drawn agreeable to it.

Again the little angle SPV which is made at the pole of the equator, and from thence contains almoſt the ſun's ſemidiameter, muſt be added to the angle ZPV at a morning's ingreſs or evening's egreſs, and ſubtracted from it at an evening's ingreſs or morning's egreſs.

GREENWICH.

For the total ingreſs.

The zenith diſtance = 85° 11′ 8″, and the angle AVB = 51° 51′ 20″.

Times of par. of long.	3′ 47″,5	or	5′ 10″,	or	6′ 14″,6
Parallax of latitude,	2 33,7	or	3 28,5	or	4 10,5
Sum	6 21,2	or	8 38,5	or	10 25,1

which ſubtract from the central time = 7^{h} 29′ 40″,5, and this ingreſs = 7^{h} 23′ 19″,3, or 7^{h} 21′ 2″, or 7^{h} 19′ 15″,4 at Greenwich.

DUBLIN.

For the firſt external contact.

Dublin is ſuppoſed to be 6° 41′ 0″ longitude weſt of Greenwich, and 53° 20′ 0″ latitude north; the zenith diſtance = 77° 56′ 58″: AVZ or BVA = 52° 23′ 46″.

Times of par. of long.	3′ 44″,8	or	5′ 6,″4	or	6′ 10″,2
Parallax of latitude	2 16,5	or	3 3,2	or	3 40,5
Sum	6 1,3	or	8 9,6	or	9 50,7

If this contact at the center by Dublin time be at 6^{h} 42′ 47″,5, then it will be at Dublin = 6^{h} 36′ 46″,2, or 6^{h} 34′ 37″,9 or 6^{h} 32′ 56″,8.

DUBLIN.

For the total ingreſs.

The zenith diſtance = 80° 43′ 40″, ZVA or BVA = 51° 9′ 48″.

Times of par. of long.	3′ 43″,2	or	5′ 4″,2	or	6′ 7″,3
Parallax of latitude,	2 34,5	or	3 28,5	or	4 12
Sum	6 17,7	or	8 32,7	or	10 19,3

which being ſubtracted from the central time of this contact, that is from $7^{h}\ 2'\ 56'',5$, it will be at Dublin = $6^{h}\ 56'\ 38'',8$, or $6^{h}\ 54'\ 23'',8$, or $6^{h}\ 52'\ 37'',2$.

QUEBEC, the metropolis of CANADA.

For the firſt external contact.

Quebec is ſuppoſed to be 70° 0′ 0″ longitude weſt of Greenwich, and 47° 12′ 0″ north latitude.

The zenith diſtance = 38° 31′ 44″, AVB = 56° 46′ 18″.

Times of par. of long.	2′ 31″,2	or	3′ 26″,1	or	4′ 9″
Parallax of latitude,	1 21	or	1 46,5	or	2 7,5
Sum	3 52,2	or	5 12,6	or	6 16,5

which being ſubtracted from the central time of this contact, which by Quebec time = $2^{h}\ 29'\ 31'',5$ in the afternoon, will make this contact at Quebec = $2^{h}\ 25'\ 39'',3$, or $2^{h}\ 24'\ 18'',9$, or $2^{h}\ 23'\ 15''$.

QUEBEC.

For the total ingreſs.

The zenith diſtance = 41° 40′ 55″, AVB = 58° 55′ 29″.

Times of par. of long.	2′ 45″,3	or 3′ 45″,3	or 4′ 32″,1
Parallax of latitude,	1 24	or 1 55,5	or 2 21
Sum	4 9,3	or 5 40,8	or 6 53,1

which ſubtract from the central time which is = 2^{h} 40′ 40′,5, and this ingreſs at Quebec will be = 2^{h} 36′ 31″,2, or 2^{h} 34′ 59″,7, or 2^{h} 33′ 47″,4.

CHARLES-TOWN, in SOUTH-CAROLINA.

For the firſt external contact.

Charles-Town is 78° longitude weſt of Greenwich, and 32° 40′ 0″ north latitude.

The zenith diſtance = 27° 36′ 36″, AVB = 77° 55′ 39″.

Times of par. of long.	2′ 11″,5	or 2′ 59″,2,	or 3′ 36″,6
Parallax of latitude,	0 24	or 0 31,5	or 0 37,5
Sum	2 35,5	or 3 30,7	or 4 14,1

which ſubtract from the central time of this contact, which is by Charles-Town time = 1^h 57′ 31″,5 in the afternoon, and it will be at Charles-Town either at 1^h 54″ 56 or 1^h 54′ 0″,8, or 1^h 53′ 17″,4.

CHARLES-TOWN.

For the total ingreſs.

The zenith diſtance = 31° 47′ 7″ BVA = 79° 40′ 33″.

Times of par. of long.	2′ 30″,4	or	3′ 25″	or	4′ 7″,6
Parallax of latititude,	0 19,5	or	0 31,5	or	0 37,5
Sum	2 49,9	or	3 56,5	or	4 45,1

which ſubtract from 2^h 17′ 40″,5 the central time of this ingreſs, and it will be at Charles-Town either at 2^h 14′ 50″,6, or 2^h 13′ 44″, or 2^h 12′ 55″,4.

Let us now compare the ſum of the times which theſe places give with one another.

GREENWICH.

1ſt. ex. con.	6′ 5″,8	or	8′ 18″	or	9′ 58″,5	—
Total ingreſs	6 21,2	or	8 38,5	or	10 25,1	—

DUBLIN.

DUBLIN.

1ſt ex. con.	6′ 1″,3 or	8′ 9″,6 or	9′ 50″,7	—
Total ingreſs	6 17,7 or	8 32,7 or	10 19,3	—

QUEBEC.

1ſt ex. contact,	3 52,2	5 12,6	6 16,5	—
Total ingreſs,	4 9,3	5 40,8	6 53,1	—

CHARLES-TOWN.

1ſt ex. contact,	2 35,5	3 30,7	4 14,1	—
Total ingreſs,	2 49,9	3 56,5	4 45,1	—

WATERLAND.

1ſt ex. contact,	4 16,8	5 51,2	7 2,2	+
Total ingreſs,	4 18,3	5 51,4	7 3,7	+

CALIFORNIA.

1ſt ex. contact,	0 30,2	0 39,7	0 47	+
Total ingreſs,	0 12,7	0 18,3	0 20	+

As the difference of the times of either contacts between Greenwich and Dublin, is not more than four or five ſeconds, when the parallax of the ſun is ſuppoſed to be 8 1-half, it is very probable that the difference of the times between Dublin and any place within Great Britain, will not exceed that ſum, and in moſt places it will be leſs.

Difference of time between Greenwich and Waterland.

1ſt ex. con.	10' 22",6 or	14' 9",2 or	17' 0",7
Total ingr.	10 39,5	14 29,9	17 28,8

Difference between Greenwich and California.

1ſt exter. contact,	6 36	8 57,7	10 45,5
Total ingreſs,	6 33,9	8 56,8	10 45,1

Difference between Greenwich and Charles Town.

1ſt exter. contact,	3 30,3	4 47,3	5 44,4
Total ingreſs,	3 31,3	4 42	5 40

Difference between Greenwich and Quebec.

1ſt exter. contact,	2 13,6	3 5,4	3 42
Total ingreſs,	2 11,9	2 57,7	3 32

NANKIN,

NANKIN, in CHINA.

The beginning of egreſs.

Nankin is ſituated about 32 degrees north latitude, and 242 longitude weſt of the meridian of Greenwich.

The zenith diſtance = 39° 10′ 54″, AVB = 50° 51′ 57″.

Times of par. of long.	2′ 22″,2 or	3′ 13″,9 or	3′ 54″,1	+
Parallax of latitude,	1 39	2 15	2 43,5	+
Sum	4 1,2	5 28,9	6 37,6	+

The time of this egreſs at the center by Nankin time is at 9^{h} 6′ 34 ,5 in the morning, and therefore it will be at Nankin 9^{h} 10′ 35″,7, or 9^{h} 12′ 3″,4 or 9^{h} 13′ 12″,1.

NANKIN, in CHINA.

Laſt external contact.

The zenith diſtance = 34° 54′ 13″, AVB = 50° 18′ 7″.

Times of par. of long.	2′ 7″,8 or	2′ 54″,1 or	3′ 30″,3	+
Parallax of latitude,	1 24	1 54	2 18	+
Sum	3 31,8	4 48,1	5 48,3	+

This contact at the center will be at 9^h 26′ 43″,5, in the morning, therefore it will be at Nankin either at 9^h 30′ 15″,3, or 9^h 31′ 31″,6 or 9^h 32′ 31″,8.

WATERLAND.

The beginning of egreſs.

The zenith diſtance = 53° 28′ 50″, AVB = 49° 9′ 30″.

Times of par. of long.	2′ 56″,4 or	4′ 0″,4 or	4′ 50″,5 —
Parallax of latitude,	2 18,2	3 9	3 48 —
Sum	5 14,6	7 9,4	8 38,5 —

This egreſs at the center by Waterland time is at 3^h 6′ 34,5 in the afternoon, therefore at Waterland it will be 3^h 1′ 19″,9, or 2^h 59′ 25″,1 or 2^h 57′ 56″.

WATERLAND.

Laſt external contact.

The zenith diſtance = 57° 45′ 27″: AVZ = 52° 22′ 45″.

Times of par. of long.	3′ 14″,4	or 4′ 25,″	or 5′ 20″,1
Parallax of latitude	1 57,	or 2 43,5	or 3 18,
Sum	5 11,4	or 7 8,5	or 8 38,1

This contact at the center by Waterland time is at 3^h 26′ 43″,5 in the afternoon; therefore at Waterland it will be at 3^h 21′ 32″,1, or 3^h 19′ 35″,3, or 3^h 18′ 5″,4.

CALIFORNIA.

Beginning of egreſs.

The zenith diſtance = 71° 7′ 36″, AVB = 85° 41′ 48″.

Times of par. of long.	4′ 33″,9	or	6′ 13″,2	or	7′ 30″,9	—
Parallax of latitude,	0 18		0 24		0 28,5	+
Difference	4 15,9		5 49,2		7 2,4	—

This egreſs at the center by California time is at 5^h 14′ 34″,5 in the afternoon; therefore at California it will be at 5^h 10′ 18″,6, or 5^h 8′ 45″,3, or 5^h 7′ 32″,1.

CALIFORNIA.

Laſt external contact.

The zenith diſtance = 75° 29′ 48″, AVB = 84° 15′ 20″.

Times

Times of par. of long.	4′ 39″,6	or	6′ 21″,	or	7′ 40″,3	—
Parallax of latitude,	0 24,0	or	0 31,5	or	0 36,	+
Difference	4 15,6	or	5 49,5	or	7 4,3	—

This contact at the center by California time is at 5^h 34′ 43″,5 in the evening; therefore at California it will be at 5^h 30′ 27″,9; or 5^h 28′ 54″, or 5^h 27′ 39″,2 +.

The ſum or difference of the times which the aforementioned places give, is at

NANKIN, in CHINA,

Beg. of egr.	4′ 1″,2	or	5′ 28″,9	or	6′ 37″,6	+
Laſt ex. con.	3 31,8		4 48,1		5 48,3	+

WATERLAND,

Begin. of egreſs,	5 14,6	7 9,4	8 38,5	—
Laſt ext. contact,	5 11,4	7 8,5	8 38,1	—

CALIFORNIA.

Begin. of egreſs,	4 15,9	5 49,2	7 2,4	—
Laſt ext. contact,	4 15,6	5 49,5	7 4,3	—

Difference between Nankin and Waterland.

Beg. of egr.	9′ 15″,8	or	12′ 38″3,	or	15′ 16″,1	—
Laſt ex. con.	8 43,2	or	11 56,6	or	14 26,4	—

Difference between Nankin and California.

Begin. of egreſs,	8 17,1	11 18,1	13 40,
Laſt ext. contact,	7 47,4	10 37,6	12 52,6

CHAP.

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CHAP XI.

Of the apparent duration of the tranſit of Venus over the ſun.

1. WHEN the parallaxes of longitude and latitude are of the ſame denomination both for the ingreſs and egreſs at the ſame place, and when they for the ingreſs retard, and they for the egreſs accelerate the tranſit at one place; whilſt at another place they for the ingreſs accelerate, and they for the egreſs retard rhe tranſit; and theſe parallaxes give the greateſt time poſſible at both places, then there will be the greateſt difference between the central duration and either apparent duration, or between the two apparent durations compared with one another.

2. The determination of the beſt places for obſerving theſe durations depends upon the nature of the tranſit, whether it be before or after the node; it will be neceſſary therefore to treat of them ſeparately, I

will begin according to our former method with the duration of the tranſit after the node.

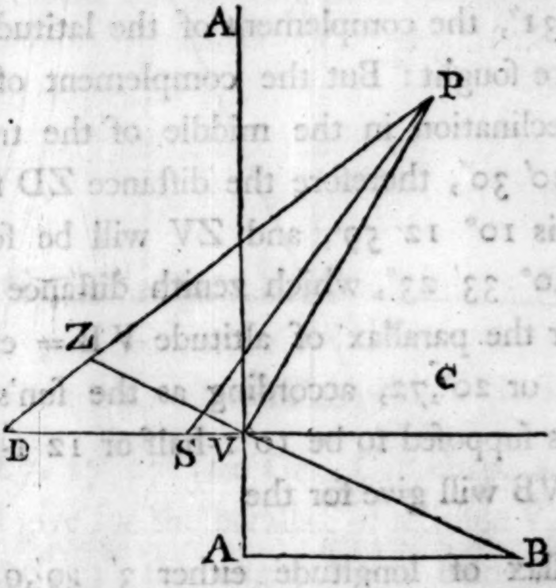
3. In order to find the beſt places for obſerving the duration of this tranſit, and where there is the greateſt retardation of the ingreſs and acceleration of the egreſs, the obſerver muſt be on the north ſide of the orbit, and on the meridian paſſing through Venus in the middle of her tranſit, that the parallaxes may be of the ſame denomination, and give the greateſt time upon the whole.

At what diſtance on the meridian on the northern ſide of the orbit the obſerver ſhould be, may be known from the following proceſſes.

Firſt, let us ſuppoſe the angle A V B, or what the parallax of latitude makes with the parallax of altitude to be 51 degrees, and ſee what diſtance and times that will give,

Then ſecondly, let us ſuppoſe the greateſt zenith diſtance from Venus on the meridian, and ſee what times that will give, and from comparing theſe with the former it will appear which is the moſt convenient place for making this obſervation.

Firſt, let A V B or Z V A be an angle of 51 degrees at the total ingreſs of the tranſit in 1761, which was after the node; the angle PVC = 75° 34′ 51″ (ſee Gen. Cal. chap. the fourth): The arch PD is ſuppoſed to be



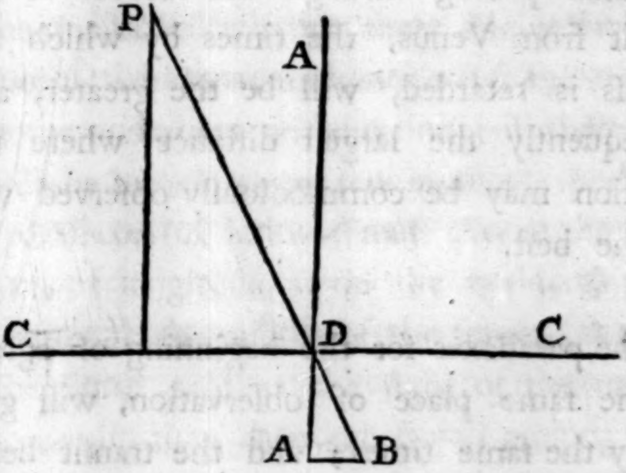
drawn from the pole of the equator through the middle of the tranſit to the orbit of Venus: The angle A V P will be 14° 25′ 9″, which added to Z V A will make Z V P = 65° 25′ 9″; and the angle ZPV by ſuppoſition is equal to half the time of the tranſit, from the total ingreſs to the beginning of egreſs, turned into degrees at the rate of 15 degrees to an hour, and increaſed by the little angle VPS, which will be 44° 45′ 6″ nearly; wherefore in the triangle ZPV the two angles at V and P are given, with the ſide VP between them which is 67° 17′ 27″ (ſee Gen. Cal. chap. 4) hence the arch PZ comes out 57° 7′ 31″, the complement of the latitude of the place ſought: But the complement of the ſun's declination in the middle of the tranſit is 67° 20′ 30″, therefore the diſtance ZD from Venus is 10° 12′ 59″, and ZV will be found to be 40° 33′ 23″, which zenith diſtance will give for the parallax of altitude VB = either 17″,15, or 20″,72, according as the ſun's parallax is ſuppoſed to be 10 1-half or 12 1-half, which VB will give for the

Parallax of longitude either 3′ 19′,9, or 4′ 1″,5 in time.

The Parallax of latitude will give 2′ 10″,5 or 2′ 35″,5 in time.

The ſum of both times is 5′ 30″,4, or 6′ 37″ for this ingreſs.

Again let us ſuppoſe the north pole to be the place of obſervation, which is nearly the greateſt diſtance that can be in tranſits after the node, where the parallaxes are of the ſame denomination; and then PV the zenith diſtance from Venus at the total ingreſs will



be 67. 17′ 27″ (ſee Gen. Cal. chap. 4) which will give for the parallax of altitude VB 24″,33 or 29″,39; the angle PVC being as before 75° 34′ 51″, PVA or AVB = 14° 25′ 9″: hence,

Parallax of long.	1′ 30″,8	or	1′ 49″,7	in time.
Parallax of lat.	4 52,5		5 55,5	
Sum	6 23,3		7 45,2	

which is greater than the ſum which the angle A V B when 51 degrees gave, by 52″,9 or 1′ 8″,2.

Hence it appears, that the larger the diſtance towards the north pole is taken on the meridian paſſing through the middle of the tranſit from Venus, the times by which the ingreſs is retarded, will be the greater, and conſequently the largeſt diſtance where the duration may be commodiouſly obſerved will be the beſt.

The parallaxes for the beginning of egreſs, at the ſame place of obſervation, will give nearly the ſame times; and the tranſit being retarded at the ingreſs, and accelerated at the egreſs, the apparent duration will be here ſhorter than that at the center by the double of thoſe times, that is at the pole either 12′ 46″,6 or 15′ 30″,4.

Now if there was any other place where the apparent duration would be as much lengthened, as this is contracted, the difference of theſe apparent durations would be double of theſe differences that is either 25′ 33″,2 or 31′ 0″,8.

But there is no place where the apparent ingreſs will be accelerated, and the apparent egreſs retarded, when the parallaxes are of the ſame denomination; nay, what is more, it is to be queſtioned whether there is any prolongation of the apparent duration of the tranſit after the node beyond the central; if there be, it will be but of a very few minutes; perhaps the parallaxes of latitude may exceed the parallaxes of longitude, upon the meridian, paſſing through the middle of the tranſit, as near to the ſouth pole as the ſight of the tranſit will permit, then their exceſs will prolong the duration, but this cannot be above two or three minutes: or again, ſhould the parallaxes of longitude exceed the parallaxes of latitude on the ſame meridian, but ſo near the north pole that the ingreſs may be obſerved in the evening, and the egreſs the next morning, their exceſs will likewiſe prolong the duration; but it is moſt probable that they will be ſhort of them.

Dr. Halley was not aware of this contrariety of parallaxes, when he recommended Port Nelſon, near Hudſon's Bay in North America, for obſerving the duration of the tranſit in 1761; he thought from the ingreſs being viſible there in the evening, and the egreſs the next morning, the apparent duration would be the largeſt that could be, and contrary to what it would be at Madraſs, Bencoolen, ot Batavia: but ſince the parallaxes of longitude and latitude there, both at the ingreſs and the egreſs, would not be of the ſame denomination in any of theſe places, the difference of any of their durations from the central, or from one another, would fall very ſhort of what was expected.

For when theſe parallaxes at the ſame place are contrary to one another, their difference will not be one fourth part of what their ſum might be; and conſequently, if the obſervation of the duration, made in any place between ten degrees north of the orbit and the north pole, on the meridian paſſing through the middle of the tranſit, will give about thirteen or fifteen minutes for the greateſt contraction of that duration when the parallaxes for the ingreſs and egreſs are of the ſame denomination: Obſervations in any other place where they are contrary to each other, will give very little more than three minutes, or three and three quarters. Hence ſhould the apparent duration be prolonged by theſe few minutes, though even this very rarely happens, the difference between theſe two apparent durations, when it is as great as poſſible, will not amount to more than ſixteen or nineteen minutes; the greateſt difference therefore of the durations of the tranſit, at two different places, will be leſs than the greateſt difference of the times of the ingreſſes or egreſſes at two different places: And ſince there muſt be two obſervations made at each place for the duration, and but one for the ingreſs or egreſs, the former muſt be more ſubject to error than the latter, and conſequently two durations of the tranſit after the node are not ſo proper ſubjects for obſervation, as two ingreſſes or egreſſes.

Aſtronomers therefore rightly determined to make their obſervations upon the apparent ingreſs or egreſs at different places, and to compare their difference with the difference that ſhould ariſe from the computed ingreſſes or egreſſes at thoſe places; becauſe this computed difference will be very little affected by any errors in the tables for the time of conjunction, or by any miſtakes concerning the central duration of the tranſit, or its ingreſs or egreſs, ſince thoſe miſtakes would be the ſame in both places.

But in my opinion the duration of the tranſit after the node ſhould always be obſerved in ſome place between the north pole, and ten degrees north from the place where the ſun is vertical at the middle of ſuch tranſit, upon the meridian paſſing through it, becauſe it is thirteen or fifteen minutes ſhorter than the central duration, which is a very notable difference, and well worthy the attention of Aſtronomers.

What hath been ſaid and applied in this chapter to the tranſit of Venus in 1761, will hold good, and may be applied to every future tranſit after the node.

CHAP.

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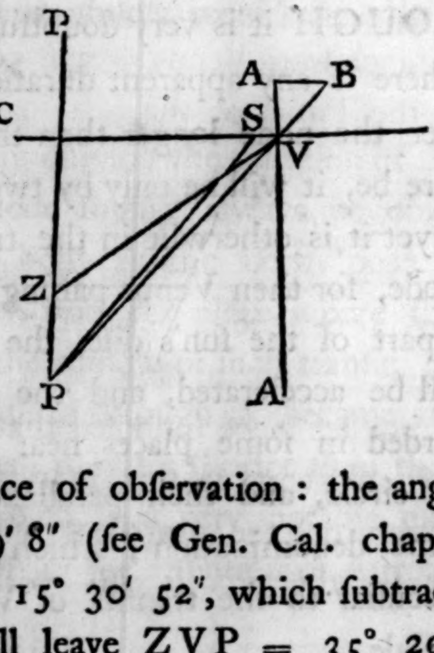
CHAP. XII.

Of the duration of the tranſit before the node.

1. THOUGH it is very doubtful whether there is any apparent duration of the tranſit after the node longer than the central, and if there be, it will be only by two or three minutes; yet it is otherwiſe in the tranſits before the node, for then Venus paſſing over the northern part of the ſun's diſc, the evening's ingreſs will be accelerated, and the morning's egreſs retarded in ſome places near or within the arctick circle, and their parallaxes will be of the ſame denomination; which is an advantage peculiar to the tranſits of Venus before the node.

2. Let us begin as we did in the laſt chapter, with the retardation of the ingreſs, and the acceleration of the egreſs, and here the obſerver muſt be on the ſouth ſide of the orbit of Venus, and on the meridian paſſing through

the middle of the tranſit: at what part he ſhould be will appear from the following proceſſes. Let A V B or Z V A be an angle of fifty-one degrees; let P P be the meridian paſſing through the middle of the tranſit; P and P the two poles of the equator; and

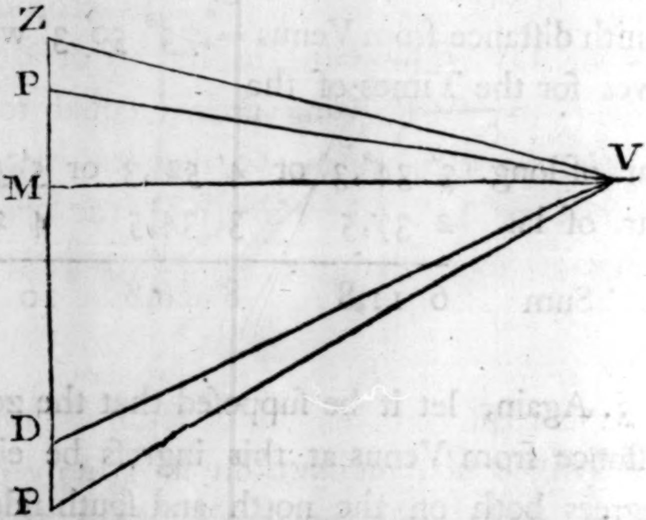


Z the place of obſervation: the angle P V C = 74° 29′ 8″ (ſee Gen. Cal. chap. 5) and P V A = 15° 30′ 52″, which ſubtracted from Z V A will leave Z V P = 35° 29′ 8″ the ſide P V = 112° 41′ 1″ (ſee the complement of P L to two right angles in Gen. Cal. chap. 5) and the angle Z P V is equal to half the time of the tranſit, from total ingreſs to the beginning of egreſs, turned into degrees, at the rate of fifteen degrees to an hour, and augmented by the little angle SPV (for which ſee Gen. Cal. chap. 5) that is Z P V = 43° 15′ 45″; therefore in the triangle Z P V, two angles and the ſide between them being given, the remaining angle and ſides will be known: The ſide PZ = 53° 35′ 9″, whoſe complement to ninety degrees = 36° 24′ 51″, and is the latitude of the place ſought, and Z V the zenith diſtance from Venus = 75° 50′ 3″ which gives for the Times of the

Par. of long.	3′ 34″,3	or	4′ 52″,3	or	5′ 52″,8
Par. of lat.	2 37,5		3 34,5		4 21
Sum	6 11,8		8 26,8		10 13,8

3. Again, let it be ſuppoſed that the zenith diſtance from Venus at this ingreſs be eighty degrees both on the north and ſouth ſides of the orbit, for the proceſs will be the ſame in both ſides.

Let P and p be the two poles of the equator, P the north pole, let V be the place of Venus at the ingreſs as ſeen from the center, let ZV and DV be eighty degrees; and ſuppoſe VM perpendicular to the meridian P p paſſing through the middle of the tranſit; then in the triangle PVM the angle MPV is as before 43° 15′ 45″. PV = 67° 18′ 59″ (ſee Gen. Cal. chap. 5.) and the angle at M is a right angle, wherefore MP comes out 60° 8′ 46″ and VM = 39° 13′ 21″. Again in the triangle MZV the ſides ZV and MV with the angle at M being given, the ſide ZM will come out 77° 2′ 50″, from which ſubtract MP, and there will remain ZP = 16° 54′ 4″, which is the diſtance of the place ſought from the north pole, or the complement of its latitude.



But if ZV be ſuppoſed to be 85 degrees, the ſide ZM will be 83° 32′ 25″, and by ſubtracting M P from it there remains Z P = 23° 23′ 39″ for the complement of its latitude north.

Again on the ſouth ſide, as the triangle V M D is the ſame with the triangle Z M V, the ſide M D being equal to M Z it will be either 77° 2′ 50″ or 83° 32′ 25″ according as ZV or DV is ſuppoſed to be 80 or 85 degrees: but M P being proved equal to 60° 8′ 46″ and P p being 180 degrees M p will be 119° 51′ 14″, wherefore p D the complement of ſouth latitude = 42° 48′ 24″ or 36° 18′ 49″.

When the complement of ſouth latitude is 42° 48′ 24″ or the zenith diſtance from Venus at the total ingreſs is 80 degrees, the times which the parallax of longitude give are

	3′ 17″,5 or	4′ 29″,2 or	5′ 25″,3
Par. of lat.	3 7,5	4 16,5	5 6
Sum	6 25	8 45,7	10 31,3
which is more by	13,2	18,9	20,5

than the ſum of the times when the angle which the parallax of altitude makes with the parallax of latitude is 51 degrees.

But when the complement of ſouth latitude is 36° 18′ 49″ or the zenith diſtance is 85 degrees, then the times which the par. of long.

give are	3′ 8″,1	or	4′ 16″,3	or	5′ 9″,1
Par. of latitude,	3 19,5		4 33		5 31,5
Sum	6 27,6		8 49,3		10 40,6
which is more by	15,8		22,5		26,8

than the ſum of the times when the aforeſaid angle is 51 degrees.

Wherefore any place on the meridian paſſing through the middle of the tranſit from the complement of ſouth latitude 36° 18′ 49″ where the zenith diſtance from Venus is 85 degrees to the complement of ſouth latitude 53° 35′ 9″, where the aforementioned angle is 51 degrees, will do very well for obſerving the total ingreſs and beginning of egreſs, though the times are the greateſt at 36° 18′ 49″ the complement of ſouth latitude for 85 degrees zenith diſtance.

The ſum of the times will be nearly the ſame at the beginning of egreſs as at this ingreſs; only this ingreſs will be retarded whilſt the egreſs is accelerated by it, ſo that the central duration will exceed the apparent duration there by twice that time: that is either by 12′ 55″,2 or 17′ 39″,6 or 21′ 21″,2.

If the place of obſervation be at 16° 54′ 4″ or 23° 23′ 39″ from the north pole on the meridian paſſing through Venus at the middle of the tranſit, and on the different ſide of the pole from what Venus is, and the total ingreſs be obſerved in the evening, and the beginning of egreſs the next morning, then the apparent duration will exceed the central duration; the parallaxes likewiſe will be of the ſame denomination, and accelerate the ingreſs and retard the egreſs.

When the complement of latitude north is 16° 54′ 4″ or the zenith diſtance from Venus is 80 degrees, then the times of the total ingreſs of the

Par. of long. are	2′ 10″,6	or	2′ 57,9	or	3′ 34″,9
Par. of latitude,	3 37,5		4 54		5 52,5
Sum	5 48,1		7 51,9		9 27,4

But when the complement of latitude north is 23° 23′ 39″, or the zenith diſtance is eighty five degrees, then

Par of long.	2′ 30″,4	or	3′ 25″	or	4′ 7″,6
Par. of lat.	3 31,5		4 45		5 43,5
Sum	6 1,9		8 10		9 51,1

The beginning of egreſs will give nearly the ſame times, only the ingreſs will be accelerated when the egreſs is retarded by them, wherefore this apparent duration will exceed the central duration by twice thoſe times, that is by 12′ 3″,8 or 16′ 20″ or 19′ 42″,2 : And conſequently the difference between the apparent durations on the north and the ſouth ſides of the orbit at 85 degrees diſtance in both places from Venus, will be the ſum of the times, viz. 24′ 59″ or 33′ 59″,6 or 41′ 3″,4.

Theſe are general calculations ſuited to all
tranſits

tranſits of Venus over the ſun before the node; and when they are applied to the meridian paſſing through the middle of any particular tranſit will determine the beſt places for obſerving the duration of it.

Thus for example, theſe latitudes upon the meridian of the tranſit which will be *June* 3, 1769, will ſhew the exact places for obſerving that duration: It unfortunately happens indeed that the ſouthern latitudes upon that meridian lie in the great ſouth ſea, where there appears to be no land, except ſome few iſlands, ſuch as *Waterland*, &c. which are very diſtant and difficult to be got at.

The moſt convenient place which the latitudes on the north part of the meridian, paſſing through the middle of the tranſit, give for obſerving the duration of this tranſit, ſeems to me, by the beſt judgment I can form from our moſt correct maps, to be *Torno* a town in *Lapland*, lying on the northern part of the *Bothnia* ſea; it hath a large and frequented haven, and therefore is eaſy to be come at; it is ſituated near 66° 3′ 0″ north latitude, and 25 degrees longitude eaſt of *Greenwich*; and conſequently the time at *Torno* is one hour and forty minutes later than *Greenwich* time, hence the middle of the tranſit at the center by *Torno* time will be at 12° 2′ 7″,5 at night; which ſhews this place to be very little diſtant from the meridian paſſing through thc middle of the tranſit at *June* 3, 1769, and its complement of latitude being 23° 57′ 0″ is very near to what we have demonſtrated to be beſt.

But any place ſuch as *Wardhus*, and other towns in *Lapland*, and within or near the Artick circle, will be very proper for obſerving the duration of this tranſit.

As *Torno* is nearly equal to the beſt place, and perhaps more commodious from its ſituation on the ſea coaſt than any other, I have made choice of it in computing the times of all the contacts for it.

TORNO.

T O R N O.

Firſt external contact.

The zenith diſtance = 84° 12′ 30″, the angle BVA = 33° 5′ 35″.

Times of par. of long.	2′ 37″,6 or	3′ 34″,9 or	4′ 19″,6	—
Parallax of latitude,	3 3,2	4 7,5	4 57	—
Sum	5 40,8	7 42,4	9 16,6	—

This central contact by *Torno* time is at 8 hours 49′ 31″,5 in the evening, and therefore at *Torno* it will be at 8ʰ 43′ 50″,7 or 8ʰ 41′ 49″,1 or 8ʰ 40′ 14″,9, according as the ſun's parallax is ſuppoſed to be 8″ 1-half, or 10′ 1-half, or 12″ 1-half.

T O R N O.

Firſt internal contact, or total ingreſs.

The zenith diſtance = 85° 33′ 55″, BVA = 31° 33′ 10″.

Times of par. of long.	2′ 31″5 or	3′ 25″,5 or	4′ 9″,3	—
Parallax of latitude,	3 30	4 45	5 42	—
Sum	6 1,5	8 10,5	9 51,3	—

This central contact is by *Torno* time at 9^{h} 9′ 40″,5 in the evening, and therefore it will be in *Torno* at 9^{h} 3′ 39″, or 9^{h} 1′ 30″, or 8^{h} 59′ 49″,2.

TORNO.

Beginning of egreſs.

The zenith diſtance = 85° 21′ 52″, AVB = 0° 44′ 1″.

Times of par. of long.	0′ 3″,7	or	0′ 5″,04	or	0′ 6″,08	+
Parallax of latitude,	4 6	or	5 33,4	or	6 35,5	+
Sum	4 9,7		5 38,44		6 41,58	+

This egreſs at the center by *Torno* time is at 2^{h} 54′ 34″,5 in the morning, and it will be in *Torno* at 2^{h} 58′ 44″,2 or 3^{h} 0′ 12″,04 or 3^{h} 01′ 16″.

TORNO.

Laſt external contact.

The zenith diſtance = 83° 59′ 26″ AVB = 2° 15′ 7″.

Times of par. of long.	0′ 11″,3	or	0′ 15″,4	or	0′ 18″,6	+
Parallax of latitude,	3 43,5		5 1,5		6 0	+
Sum	3 54,8		5 16,9		6 18,6	+

This contact at the center by *Torno* time is at 3^{h} 14′ 43″,5 in the morning, and therefore it will be in *Torno* at 3^{h} 18′ 38″,3 or 3^{h} 20′ 0″,4 or 3^{h} 21′ 2″,1.

The duration of this tranſit from firſt to laſt external contact at *Torno* exceeds the central duration by 9′ 35″,6 or 12′ 59″,3 or 15′ 35″,2.

From total ingreſs to the beginning of egreſs exceeds it by 10′ 11″,2 or 13′ 48″,9 or 16 32″,8.

The duration of the tranſit from firſt to laſt external contact at *Waterland* falls ſhort of the central duration by 9′ 28″,2 or 12′ 59″,7 or 15′ 40″,3.

From total ingreſs to the beginning of egreſs falls ſhort of it 9′ 32″,9 or 13′ 0″,8 or 15′ 42″,2.

The duration of the tranſit from firſt to laſt external contact at *California* falls ſhort of the central duration by 4′ 45″,8 or 6′ 29″,2 or 7′ 51″,3.

From total ingreſs to the beginning of egreſs falls ſhort of it by 4′ 28″6 or 6′ 7″,5 or 7′ 22″,4.

The difference therefore of the durations of the tranſit between *Torno* and *Waterland* from firſt to laſt external contact is 19′ 3″,8 or 25′ 59″ or 31′ 15″,5.

From total ingreſs to the beginning of egreſs at thoſe places will be 19′ 44″,1 or 26′ 49″,7 or 32′ 15″.

The difference likewiſe of the durations between *Torno* and *California* from firſt to laſt external contact will be 14′ 21″,4 or 19′ 28″,5 or 23′ 26″,5.

From total ingreſs to the beginning of egreſs at thoſe places will be 14′ 39″,8 or 19′ 56″,4 or 23′ 55″,2.

To conclude : In the five laſt chapters are ſhewn the latitudes and longitudes of the beſt places for obſerving the ingreſs or egreſs or duration of any tranſit of Venus over the ſun, either before or after the node, with the difference of their times from thoſe at the center; what appertaineth to tranſits after the node is treated of in the eighth and eleventh chapters, and to tranſits before the node in the ninth, tenth, and twelveth chapters.

The ſame latitudes are in all tranſits upon the ſame parallel circle to the equator; but the ſame longitudes are upon different meridians in different tranſits, becauſe they are meaſured from the meridian that paſſes through the ingreſs or egreſs of every particular tranſit, which is therefore a variable meridian, and changes with the place of that ingreſs or egreſs: All which have been exemplified at large for tranſits after the node in the deſcription of that which was *June* 6th, 1761, and for tranſits before the node of that which will be *June* 3d, 1769.

OF

The TRANSITS of MERCURY

Over the SUN.

CHAP. I.

1. THOUGH the difference between the parallaxes of Mercury and the ſun, or what is generally called the parallax of Mercury from the ſun, is leſs than the parallax of the ſun, yet it may be found more eaſily by means of the tranſits of this planet over the ſun; which have this peculiar advantage that from the ſlowneſs of Mercury's motion, and alſo of the ſun or earth in its orbit, they aſford a ſufficient quantity of time for aſcertaining this parallax very nearly: How nearly it may be aſcertained, or what portion of time will be given by theſe tranſits for meaſuring this parallax is the ſubject

jeƈt of our preſent enquiry, and humbly ſubmitted to the conſideration of aſtronomers.

II. The orbit of Mercury is the moſt eccentrical of all the primary planets, and the line of its apſides paſſes very nearly through its nodes with the eccliptic; there is therefore a wide difference between the diſtances of this planet from the earth at its different nodes: At the aſcending node, its diſtance from the earth to that from the ſun will be almoſt two to one: But at the deſcending node it will be very little more than as five to four. Hence the tranſits at the aſcending node will not give much more than a third part of the time for meaſuring the parallax of Mercury from the ſun, which the tranſit at the oppoſite node will give.

As this then will be generally too ſmall to anſwer the intended purpoſe, we ſhall enter no farther into it than is neceſſary for explaining this ſubjeƈt, and illuſtrating the tranſit at the deſcending node, which if I am not miſtaken will when accurately obſerved by good inſtruments, and at proper places, afford ſufficient time for determining the parallax of the ſun within half a ſecond of a degree.

CHAP. II.

Of the tranſit of Mercury at the aſcending node.

ASTronomers have fixed the place of the aſcending node of Mercury's orbit with the eccliptic as ſeen from the ſun, at 16 degrees and 44 minutes in Taurus.

As there will be a tranſit of Mercury over the ſun at this node in the year 1769, we will give a brief detail of it, and of the times which it gives for determining the parallax, by way of example for illuſtrating theſe kinds of tranſits.

The true time of this conjuction of Mercury with the ſun will be as given by the tables, on the 9th day of *November*, 1769, and at ten minutes paſt ten at night.

The ſun's place at the ſame time in the ecliptic as ſeen from the earth will be in ſcorpio at 17° 49′ 49″.

Therefore the tranſit will be after Mercury hath paſſed the node and the diſtance of the ſun beyond it will be 1° 5′ 49″.

The angle which the orbit makes with the eccliptic is 6° 54′ 0″.

Diſtance of Mercury from the ſun as given by Mr. *Whiſton* 31365.

Diſtance of Mercury from the earth as given by Ditto 67591.

Diſtance of the earth from the ſun 98956.

Equated horary motion of Mercury as ſeen from the ſun 0° 15′ 9″.

Equated horary motion of the ſun in the eccliptic 0° 2′ 31″.

Difference of the two horary motions 0° 12′ 38″.

Sun's ſemidiameter 0° 15′ 50″.

Mercury's apparent ſemidiameter on the diſc of the ſun 0° 0′ 7″,7.

Heliocentrick latitude of Mercury from the eccliptic 0° 7′ 58″.

Geocentrick latitude of Mercury from the eccliptic 0° 3′ 44″.

Geocentrick horary motion of Mercury on the ſun 0° 5′ 52″.

Which makes the common multiplyer for turning minutes of degrees into minutes of time to be 10″,22.

The angle of the apparent node or which the apparent tranſit line makes with the apparent eccliptic 3° 11′ 51″.

Suppoſe the ſun's parallax to be nine ſeconds of a degree, then Mercury's Parallax at this node will be 13″,1 and its parallax from the ſun 4″,1: and conſequently the parallax of altitude at 80 degrees from the zenith will be 4″,03: hence when the angle which the line expreſſing the parallax of altitude makes with the line expreſſing the parallax of latitude is 51 degrees: the parallax of longitude = 3″,138 which in time are 32″,07: that of latitude is 2″ giving 1″,3 which in time is 13″,28 +; the ſum of both times is 45″,35. As this will be the exceſs of the time in the morning's obſervation of the firſt external contact of *Mercury* with the ſun on the 9th of *November* 1769, above the central time; there will be the ſame defect from it in the evening's obſervation of the ſame contact at the beſt place, and at the ſame elevation above the horizon. Therefore twice this ſum of 45″,35 which is 0° 1′ 30″,7 will be the greateſt difference of time that can be between the obſervations of this contact at the two beſt places for obſerving it.

But this is a very little more than a third part of the time that will be given from the like obſervations at the deſcending node, as will appear from the following calculations of it.

CHAP. III.

Of the tranſits of Mercury at the deſcending node.

THE place of this node as ſeen from the ſun is in Scorpio 16° 44′ 0″.

The angle of Mercury's orbit with the ecliptic 6° 54′ 0″.

The relative diſtance of Mercury from the ſun 45308.

Relative diſtance of Mercury from the earth at this conjunction 55699.

Relative diſtance of the earth from the ſun 101007.

Equated horary motion of Mercury on the ſun is 0° 7′ 13″.

Equated horary motion of the ſun in the eccliptic 0° 2′ 25″.

Difference of the two horary motions 0° 4′ 48″.

The ſun's ſemidiameter 0° 15′ 50″.

Mercury's apparent ſemidiameter at this conjunction 0° 0′ 9″,56.

The geocentrick horary motion of Mercury 0° 3′ 56″.

Which makes the common multiplyer for turning minutes of a degree into minutes of time to be 15,254.

Let us for example take the tranſit of Mercury over the ſun which will be at the deſcending node in 1799, *May* 7th, = 2ʰ 34′ new ſtile, according to Mr. *Whiſton*'s tables.

The place of the node as ſeen from the earth is 1ˢⁱᵍⁿ 16° 44′ 0″.

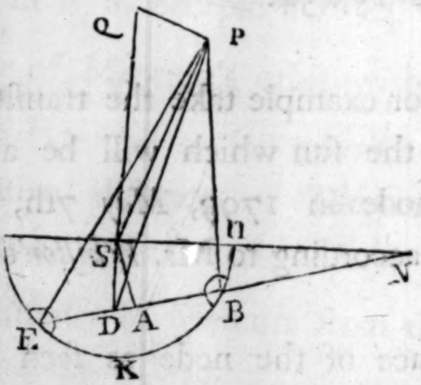
Place of the ſun 1799, *May* 7th, 2ʰ 34′ is 1ˢⁱᵍⁿ 16° 58′ 6″.

Sun's diſtance ſrom the node at the conjunction 14′ 6″.

The geocentrick latitude of Mercury from the eccliptic 3′ 3″.

The angle of the apparent node is 5° 36′ 29″.

Let M n K repreſent half the ſun's diſc, and as this tranſit is after the node let QSD be drawn from the pole of the eccliptic Q through the center of the ſun S to the tranſit line at D, which is therefore perpendicular to the eccliptic line m n N. Let SA be drawn perpendicular to the tranſit line E N. Then in the triangle S D N, the angle at S is a right angle; DNS = 5° 36′ 29″; and S D the geocentrick latitude of Mercury from the ſun at the conjunction is 0° 3′ 3″. The triangles SDN and DAS if conſidered as plane triangles are ſimilar: wherefore S A comes out 0° 3′ 2″, and E A = 15′ 22″,6 in degrees, which is 3^h 54′ 33″,3 in time, which is half the internal tranſit, and conſequently the total duration of it is 7^h 49′ 6″,6 D A = 0′ 17″,7 in degrees, which are 4′ 29″,9 in time.



Let P be the pole of the equator, draw PE, PS, PD, and PB and PQ. Then in the triangle PSQ the three ſides are given, that is PQ the diſtance of the two poles = 23° 29′ 0″ QS = 90° 0′ 0″ and P S is the complement of the ſun's declination at the time of conjunction which is = 73° 3′ 54″; wherefore the angle QSP will come out 16° 34′ 46″.

Again in the triangle P S D the two ſides P S and S D with the angle P S D being given the ſide P D will be found to be 73° 6′ 48″, and the angle QD P = 16° 34′ 13″, the angle P D N = 67° 49′ 18″, the angle P B N = 67° 53′ 42″, the angle P E N = 67° 44′ 54″: P B = 73° 1′ 4″, P E = 73° 12′ 37″.

As this is a tranſit after the node, we will for example's ſake take another which was before this deſcending node, viz. that in 1753, *May* 5, 19^{h} 20′ by *Greenwich* time according to Mr. *Whiſton*'s tables.

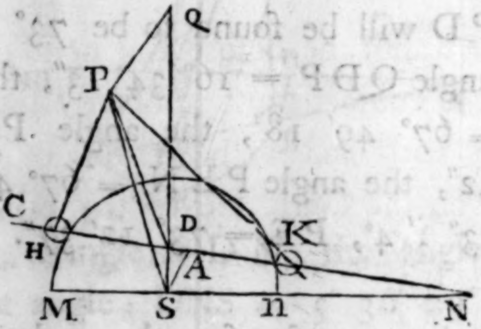
The ſun's place at that time was in $1^{\text{ſign}}$ 15° 49′ 45″.

Mercury's diſtance before the node at this conjunction 0° 54′ 15″.

Geocentrick latitude of Mercury from the eccliptic 0° 5′ 22″.

The angle of the apparent node 5° 36′ 29″.

Let M H K n be half the ſun's diſc, and as this tranſit is before this node H D K N may ſtand for the tranſit line and N the node,



let P S, P D, P Q, S Q, S A, be drawn as before, only now let P H and P K be drawn to the external contacts at H and K: Then SA will be found to be 5′ 20″, HA is 15′ 4″ in degrees, which is 3^{h} 49′ 49″ in time; and the duration of the tranſit through H K = 7^{h} 39′ 38″ PS the complement of the ſun's declination 73° 23′ 15″.

Hence QSP = 16° 50′ 30″: QDP = 16° 50′ 58″: PHC = 67° 33′ 51″: PDC = 67° 31′ 43″: PKC = 67° 27′ 21″: PH = 73° 15′ 34″: PD = 73° 18′ 7″: PK = 73° 24′ 8″.

CHAP. IV.

Of the beſt places for obſerving the tranſits of Mercury both before and after the deſcending node.

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IF the tranſit be before the node; the firſt external contact muſt be obſerved as appears from the 10th and 11th ſections of the ſixth chapter, on the ſouth ſide of the orbit in the morning, and on the north ſide in the afternoon.

And the laſt external contact muſt be obſerved on the north ſide in the morning, and on the ſouth ſide in the afternoon.

The latitudes and longitudes are as follow:

For obſerving the morning's firſt external contact, Latitude ſouth, 51° 4′ 9″.

Longitude weſt of the meridian paſſing thro' this contact, 48° 32′ 3″.

For obſerving the evening's firſt external contact.

Latitude north, 61° 25′ 45″.

Long. eaſt of the ſame meridian, 100° 4′ 11″.

For obſerving the morning's laſt exter. contact.

Latitude north, 18° 28′ 42″.

Longitude weſt of the meridian paſſing through this contact, 84° 45′ 10″.

For obſerving the evening's laſt external contact.

Latitude ſouth, 12° 34′ 35″.

Long. eaſt of the ſame meridian, 75° 23′ 40″.

If the tranſit be after the node then the firſt external contact muſt be obſerved, as appears from the 6th and 7th ſections of the ſixth chapter, on the north ſide of the orbit in the morning, and on the ſouth ſide in the afternoon.

And the laſt external contact muſt be obſerved on the ſouth ſide in the morning, and on north ſide in the afternoon.

The reſpective latitudes and longitudes are as follow:

For obſerving the morning's firſt exter. contact.

Latitude north, 19° 40′ 0″.

Longitude weſt of the meridian paſſing thro' that contact, 85° 8′ 0″.

For obſerving the evening's firſt external contact.

Latitude ſouth, 13° 40′ 0″.

Long. eaſt of the ſame meridian, 74° 56′ 0″.

For obſerving the morning's laſt external contact.

Latitude ſouth, 50° 30′ 0″.

Longitude weſt of the meridian paſſing thro' that contact, 49° 40′ 0″.

For obſerving the evening's laſt external contact.

Latitude north, 60° 33′ 0″.

Long. eaſt of the ſame meridian, 99° 29′ 0″.

Theſe and the preceding latitudes and longitudes of places for obſerving the contacts of Mercury both before and after the deſcending node are general, and will ſerve for the internal contact and beginning of egreſs, and not only of theſe aforementioned particular tranſits of Mercury over the ſun, but of all other at this deſcending node: The firſt latitudes and longitudes for any ingreſs or egreſs before the node, and the laſt for any after it.

CHAP. V.

Of the times which the obſervations of the ſame contact at different places give.

LET an obſervation be made in the morning, when the ſun is 80 degrees from the zenith of any ingreſs of Mercury upon the ſun; and where the angle which the parallax of altitude makes with the parallax of latitude is fifty one degrees.

If the parallax of the ſun be ſuppoſed eight ſeconds of a degree, then the parallax of Mercury will be 14″,5, and that of Mercury from the ſun will be 6″,5.

Parallax of longitude = 4″,98 in degrees, and 1′ 15″,9 in time.

Parallax of latitude = 4″,033.

If the parallax of the ſun be nine ſeconds then the parallax of Mercury will be 16″,32, and of Mercury from the ſun 7″,32.

Parallax of longitude = 5″,6 in degrees, and 1′ 25″,4 in time.

Parallax of latitude = 4″,53.

If the parallax of the ſun be ten ſeconds then the parallax of Mercury = 18″,13, and of Mer-from the ſun = 8″,13.

Parallax of longitude = 6″,22 in degrees, and 1′ 34″,9 in time.

Parallax of latitude = 5″.

If the obſervation be made of the total ingreſs of Mercury upon the ſun which will be *May* 7, 1799, with the conditions deſcribed above:

Then when the ſun's parallax = 8″ the parallax of latitude being 4″,033 it will give only 11″,28 in time, which being added to 1′ 15″ the time which the parallax of longitude gives will make 1′ 26″,28.

When the ſun's parallax is nine ſeconds the parallax of latitude being 4″,53 it will give only 12″,8 in time, which being added to 1′ 25″,4 the time the parallax of longitude gives the ſum will be 1′ 38″,2.

When the ſun's parallax is ten ſeconds the parallax of latitude being 5″ will give only 14″,18 in time, which being added to 1′ 34″,9 the time the parallax of longitude gives, the ſum will be 1′ 49″,08.

As the evening's obſervation at the proper place of this contact will give the ſame time in exceſs as this is in defect from the central time, the difference between theſe two obſervations will be double of the afore-mentioned times, that is. 2′ 52″,5, or 3′ 16″,4, or 3′ 38″,16.

There will be likewiſe the ſame difference of time nearly between the like obſervations of any other contact in this tranſit.

The difference of time between eight and nine ſeconds of the ſun's parallax is 23″,9, between nine and ten ſeconds is 21″,7, and between eight and ten is 45″,6.

The moſt commodious places for obſerving the total ingreſs of Mercury upon the ſun, which will be *May* 7, 1799, according to the latitudes and longitudes deſcribed above, will be for the Morning's obſervation at *Jamaica*, *Hiſpaniola*, or any of the leeward iſlands.

For the evening's obſervation of the ſame ingreſs, the iſland of *Java*, or the moſt ſouthern part of the iſland of *Sumatra* in the *Eaſt-Indies*.

For the morning's obſervation of the beginning of egreſs, the place agreeing with the aforeſaid latitudes and longitudes is in the *Great South Sea*, where there is no known land near it.

For the evening's obſervation of the ſame egreſs, any of the northern parts of *Scotland*, or *Denmark*, or the ſouthern parts of *Norway*.

If there had been obſervations made upon the firſt external contact at the tranſit of Mercury,

cury, which was at the deſcending node, *May* 5, 1753, with the conditions deſcribed above, then upon the ſuppoſition of the ſun's parallax being eight ſeconds,

The parallax of latitude = 4″,033 would have given 30″,5 in time.

And parallax of longitude = 4′,98 in degrees, or 1′ 15″,9 in time.

Sum of both = 1′ 46″,4.

If the ſun's parallax was nine ſeconds, parallax of latitude being = 4″,53, would be 33′,5 in time.

Parallax of longitude = 5″,6 in degrees, or 1′ 25″,4 in time.

Sum of both = 1′ 58″,9.

If the ſun's parallax was ten ſeconds, parallax of latitude being = 5″ would give 35″,08 in time.

Parallax of longitude = 6″,22 in degrees, or 1′ 34″,9 in time.

Sum of both = 2′ 9″,98.

Hence the difference of the two obſervations upon the ſame contact would be double of theſe times, that is 3′ 32″,8 or 3′ 57″,8 or 4 19″,96.

The difference of time between 8″ and 9″ of the ſun's parallax would have been here 25″; between 9″ and 10″ = 22″,16, and between 8″ and 10″ = 47″,16.

Whether the aſtronomical tables of Mercury's tranſits will give the time of any ingreſs or egreſs for the center of the earth, and obſervations upon the ſurface of it, will give the time of the ſame ingreſs or egreſs there ſo exact that the difference of two minutes or leſs may be depended on, muſt be left to the judgment of aſtronomers: but it ſeems to me very clear that two obſervations of the ſame ingreſs or egreſs, at two different places, may be made with ſuch accuracy as to give the difference of four minutes within a few ſeconds from each other; and if ſo then this difference will be ſufficient to determine the ſun's parallax within half a ſecond of a degree.

THE END.

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